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Creators	Mathys, G., Holdsworth, Daniel Luke and Kurtz, Donald Wayne

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Long-period Ap stars discovered with TESS data: Cycles 3 and 4

G. Mathys[1], D. L. Holdsworth[2,3], and D. W. Kurtz[4,2]

[1] European Southern Observatory, Alonso de Cordova 3107, Vitacura, Santiago, Chile
e-mail: gmathys@eso.org
[2] Jeremiah Horrocks Institute, University of Central Lancashire, Preston PR1 2HE, UK
[3] South African Astronomical Observatory, PO Box 9, Observatory 7935, Cape Town, South Africa
[4] Centre for Space Physics, North-West University, Mahikeng 2735, South Africa

ABSTRACT

One of the most challenging aspects of the Ap stars is the extreme differentiation of their rotation periods, which span more than five orders of magnitude. The physical origin of this differentiation remains poorly understood. The consideration of the most slowly rotating Ap stars represents a promising approach to gain insight into the processes responsible for the rotational braking to which the Ap stars are subject. However, historically, the study of these stars focused primarily on the most strongly magnetic among them. This bias introduced an ambiguity in the conclusions that could be drawn, as it did not allow the distinction between the rotational and magnetic effects, nor the investigation of possible correlations between rotational and magnetic properties. We previously showed that the identification of super-slowly rotating Ap (ssrAp) star candidates (defined as Ap stars that have rotation periods $P_{\rm rot} > 50$ d) through systematic exploitation of the available TESS photometric observations of Ap stars is an effective approach to build a sample devoid of magnetic bias. This approach rests on the presence of brightness spots on the surface of Ap stars that are not distributed symmetrically about their rotation axes and show long-term stability, hence are responsible for photometric variations with the stellar rotation period. In our previous analyses of TESS Cycle 1 and Cycle 2 data, we interpreted the Ap stars showing no such variability over the 27-d duration of a TESS sector as being ssrAp star candidates. Here, we applied the same approach to TESS Cycle 3 and Cycle 4 observations of Ap stars. We show, however, that two issues that had not been fully appreciated until now may lead to spurious identification of ssrAp star candidates. On the one hand, a considerable fraction of the Ap stars in the existing lists turn out to have erroneous or dubious spectral classifications. On the other hand, the TESS data processing may remove part of the variability signal, especially for stars with moderately long periods (20 d $\lesssim P_{\rm rot} \lesssim$ 50 d). After critical evaluation of these effects, we report the identification of 25 new ssrAp star candidates and of eight stars with moderately long periods. Combining this list with the lists of ssrAp stars from Cycles 1 and 2 and with the list of ssrAp stars that were previously known but whose lack of variability was not detected in our study, we confirmed at a higher significance level the conclusions drawn in our earlier work. These include the lower rate of occurrence of super-slow rotation among weakly magnetic Ap stars than among strongly magnetic ones, the probable existence of a gap between ~2 and ~3 kG in the distribution of the magnetic field strengths of the ssrAp stars, and the much higher rate of occurrence of rapid oscillations in ssrAp stars than in the whole population of Ap stars. The next step to gain further understanding of the ssrAp stars will be to obtain high-resolution spectra of those for which such observations have not been made yet, to constrain their rotation velocities and their magnetic fields.

Key words. stars: chemically peculiar – stars: magnetic field – stars: oscillations – stars: rotation

1. Introduction

More than a century ago, Ludendorff (1906) reported that a number of spectral lines in the spectrum of α^2 CVn (=HD 112413) show intensity variations. Belopolsky (1913) then found that these variations are periodic and derived an approximate value of their period, $P = 5{.}^{\rm d}50$. In the spectrum of α^2 CVn, there are spectral lines that appear more intense or weaker than those of the majority of the stars of spectral type A. Such anomalies were observed in a fraction of the A stars, which were assigned the spectral type Ap accordingly in the Harvard classification ('p' stands for 'peculiar'). It was subsequently found that at least a fraction of the Ap stars display periodic line intensity variations such as those of α^2 CVn. Such variability was recognised as typical of Ap stars, and α^2 CVn became the prototype of the class. Even now Ap stars are frequently referred to as α^2 CVn variables.

The discovery by Babcock (1947) of a magnetic field in the Ap star HD 118022 (=78 Vir) quickly led to the realisation that such fields were ubiquitously present in Ap stars (see Babcock

1958 and references therein). The star HD 125248 (=CS Vir) became the first Ap star in which the periodic variability of the magnetic field was established (Babcock 1951). Whether this variability, which is now known to be characteristic of Ap stars in general, was intrinsic (due to 'magnetic oscillations') or of geometric origin was unclear at first. Arguments supporting the latter interpretation were presented by Deutsch (1956), who developed the oblique rotator model, according to which the observed magnetic and spectral variations result from the changing aspect of the visible hemisphere of a rotating star whose surface presents inhomogeneities that are not symmetric about the rotation axis.

New light was shed on this issue by the discovery of a large population of Ap stars with resolved magnetically split lines, as a result of an extensive, systematic search project carried out by Mathys and collaborators (Mathys et al. 1997; Mathys 2017 and references therein). The resolution of the spectral lines of these stars into their magnetic components results from the combination of their strong magnetic fields (with intensities of several kilogauss) and of their low projected equatorial velocities $v \sin i$

(of a few km s^{-1}). Their identification was based on these properties. For most of them, constraints on their variation periods were derived only in a second step, from multi-epoch magnetic measurements. On a statistical basis, one must expect the majority of the stars with small values of $v \sin i$ to be genuine slow rotators (as opposed to being seen at a low inclination angle i of the rotation axis to the line of sight). On the other hand, of the 50 Ap stars with resolved magnetically split lines in the sample of Mathys (2017) for which reliable values of the variation period P, or lower limits thereof, were available, 31 (or 62%) had $P > 30$ d. Thus, stars selected on the basis of the sharpness of their spectral lines, and hence on their probable slow rotation, tend in their majority to have long variation periods. This is the reverse argument of that of Preston (1970), according to which stars selected for their long variation periods have a low projected equatorial velocity. It strengthens the evidence that all Ap stars, regardless of the length of their periods, owe their variability to the change of aspect of their visible hemisphere over a rotation cycle, as interpreted in the framework of the oblique rotator model. Within this context, the magnetic, spectral and photometric variations occur with the rotation period, and the surface inhomogeneities remain stable over very long timescales, generally covering many rotation periods.

An unexpectedly large number of Ap stars showing resolved magnetically split lines in the visible were discovered[1]. This, in turn, led to the realisation of the existence of a much larger population of very slowly rotating Ap stars than was previously estimated. It also became apparent that Ap stars with rotation periods of the order of 300 yr must definitely exist, and that some Ap stars may even have much longer periods (Mathys 2017). On the other hand, extensive studies of large samples of Ap stars based on ground- and space-based photometric surveys have confirmed that the vast majority of Ap stars have rotation periods between 1 and 10 days (Wraight et al. 2012; Bernhard et al. 2015a,b, 2020; Hümmerich et al. 2016, 2018; Netopil et al. 2017; Labadie-Bartz et al. 2023). With the shortest periods known of the order of 0.5 d and the longest ones set to be of the order of 300 yr at least, the rotation rates of Ap stars range over more than five orders of magnitude. Understanding how such a large differentiation is achieved in stars whose other physical properties appear similar represents a major challenge. In this context, achieving a better characterisation of the extreme slow rotation tail of the period distribution must be expected to provide some of the most valuable constraints to guide theoretical developments. This is the main motivation underlying studies such as the one presented here.

Mathys (2020) introduced the nomenclature super-slowly rotating Ap (ssrAp) stars to refer to the group of the longest period Ap stars. By definition, the members of this group have rotation periods $P_{rot} > 50$ d. While this definition involves a certain degree of arbitrariness, it may be noted that for an Ap star with $P_{rot} = 50$ d observed equator-on, the thermal broadening of the spectral lines of Fe (which are often used to diagnose various physical properties, such as the magnetic field) is of the same order as their rotational broadening. The latter is also close

to, or slightly below, the resolution limit ($R = \Delta\lambda/\lambda \sim 10^5$) of most high-resolution spectrographs currently used for Ap star studies.

The main limitation to the knowledge of the distribution of the rotation periods of the most slowly rotating Ap stars is the time interval over which relevant observations of each of them has been obtained. In most cases, data sampling a full rotation cycle have not been acquired yet. In fact, accurate determination of the value of a rotation period requires measurements covering substantially more than one rotation cycle to be considered. Mathys (2020) compiled a list of the ssrAp stars for which accurate periods had been derived from the analysis of data suitably distributed over a long enough time span. Furthermore, in a number of cases, meaningful lower limits of the periods of ssrAp stars can be set from the consideration of observations sampling only a fraction of a rotation period. Table 1 presents a list of the ssrAp stars that are presently known either on the basis of accurately determined period values or on reliably established period lower limits. The former part represents an updated version of Table 1 of Mathys (2020), to which one recent period determination was added.

While we tried to be as complete as possible in compiling this list, we may have overlooked the odd ssrAp star with a published period value or lower limit. However, we have also deliberately omitted several stars from the list for which long periods or limits thereof have been proposed in the literature, whenever critical evaluation suggests that the published values are not reliable enough. In particular, we did not include HD 95699 (=CD−416291) whose published period ($P = 57^d176$; Koen & Eyer 2002) may be of instrumental origin; HD 110066 (=HR 4816) for which the period values proposed by Adelman (1981), $P = 13.5$ yr or $P = 27$ yr, were not confirmed by more recent observations (Pyper & Adelman 2017); or HD 150562 (=CoD−4811127), for which we could not confirm, on the basis of our own frequency analysis of the magnetic data of Giarrusso et al. (2022), the value $P_{rot} = 2100$ d of the rotation period derived by these authors. We do not regard either the values $P_{rot} = 4000$ d or $P_{rot} = 9370$ d that they proposed for HD 29578 as fully convincing, but we agree that the former represents a reliable lower limit of the rotation period. On the other hand, in those cases for which the observations obtained until now do not adequately sample a full variation cycle, the list contains only stars that show definite variability and for which a period shorter than 50 d can be unambiguously ruled out. Accordingly, stars such as HD 92499 (=CoD−42 6407), HD 117290 (=CoD−48 8252) or HD 213637 (=BD−20 6447), for which tentative lower limits of the period are given in Tables 1 and 2 of Mathys (2017), are not included in Table 1. Neither is HD 75445 (=CD−38 4907), whose variability remains to be established (Giarrusso et al. 2022).

The table contains 34 stars whose rotation period has been accurately determined, and ten stars for which the observations obtained until now lend themselves to setting a meaningful lower limit of the period. The longest period whose value could be accurately determined until now is that of HD 50169, $P_{rot} = 10\,600$ d. Among the stars of Table 1 for which only a lower limit of the period could be derived, only one, HD 29578, may possibly have a significantly shorter period. This illustrates one of the difficulties of characterising the slow rotation tail of the period distribution: Ap stars in general have not been systematically studied yet over a long enough time interval to cover a full variation cycle of the most slowly rotating ones. For a more detailed discussion of this fundamental limitation, readers can refer to Mathys et al. (2019a).

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[1] More were found showing resolved magnetically split lines in the infrared (Chojnowski et al. 2019), but they are of lesser relevance in the context of this study. As the Zeeman effect depends on the wavelength quadratically while the wavelength dependence of the Doppler effect is only linear, for a given magnetic field strength, magnetically split lines can indeed be resolved in faster rotating stars in the infrared than in the visible.

Table 1. Known ssrAp stars.

HD/HDE	Other ID	Spectral type	$P_{\rm rot}$ (d)	Reference	B_0 (kG)	roAp
			Stars with accurately determined periods			
50169	BD−1 1414	A3p SrCrEu	10600	Mathys et al. (2019a)	5.1	
9996	HR 465	B9p CrEuSi	7937	Metlova et al. (2014)	4.7	
965	BD−0 21	A8p SrEuCr	6030	Mathys et al. (2019b)	4.3	
166473	CoD−37 12303	A5p SrEuCr	3836	Mathys et al. (2020a)	7.1	roAp
94660	HR 4263	A0p EuCrSi	2800	Mathys (2017)	6.2	
187474	HR 7552	A0p EuCrSi	2345	Mathys (1991)	5.4	
[a]	CPD−62 2717	B8 [b]	1765	Chojnowski et al. (2023)	9.9	
59435	BD−8 1937	A4p SrCrEu	1360	Wade et al. (1999)	3.0	
18078	BD+55 726	A0p SrCr	1358	Mathys et al. (2016)	3.4	
93507	CPD−67 1494	A0p SiCr	556	Mathys et al. (1997)	7.2	
2453	BD+31 59	A1p SrEuCr	518.2	Pyper & Adelman (2017)	3.7	
51684	CoD−40 2796	F0p SrEuCr	371	Mathys (2017)	6.0	
61468	CoD−27 4341	A3p EuCr	322	Mathys (2017)	6.8	
110274	CoD−58 4688	A0p EuCr	265.3	Freyhammer et al. (2008)	4.0	
69544	CD −50 3225	B8p Si	236.5	Bernhard et al. (2015a)		
188041	HR 7575	A6p SrCrEu	223.78	Mathys (2017)	3.7	
221568	BD+57 2758	A1p SrCrEu	159.10	Pyper & Adelman (2017)		
66821	CPD−54 1483	A0p Si	154.9	Bernhard et al. (2015a)		
116458	HR 5049	A0p EuCr	148.39	Mathys (2017)	4.7	
126515	BD+1 2927	A2p CrSrEu	129.95	Mathys et al. (1997)	12.7	
340577	BD+25 4289	A3p SrCrEu	116.7	Hümmerich et al. (2016)		
263361	BD+19 1468	B9p Si	88.9	Bernhard et al. (2015a)		
149766	CD−51 10384	B8p Si	87.3	Bernhard et al. (2015a)		
[c]	BD−7 1884	SrCrEu	75.97	Hümmerich et al. (2016)		
158919	CPD−65 3465	B9p Si	75.72	Bernhard et al. (2015a)		
102797	CPD−75 757	B9p Si	75.02	Koen & Eyer (2002)		
8441	BD+42 293	A2p Sr	69.51	Pyper & Adelman (2017)		
5797	BD+59 163	A0p CrEuSr	68.05	Dukes & Adelman (2018)		
123335	HR 5292	Bp He wk SrTi	55.215	Hensberge et al. (2007)		
103844	CPD−75 767	A0p Si	52.95	Bernhard et al. (2015a)		
200311	BD+43 3786	B9p SiCrHg	52.0084	Wade et al. (1997)	8.4	
256476	TYC 736-1842-1	A2p Si	51.23	Hümmerich et al. (2016)		
184471	BD+32 3471	A9p SrCrEu	50.8	Kudryavtsev & Romanyuk (2012)		
87528	CPD−62 1415	B8/9 III [d]	50.35	Bernhard et al. (2015a)		
			Stars with well-established period lower limits			
201601	γ Equ	A9p SrEu	≳35 500	Bychkov et al. (2016)	3.9	roAp
213258	BD+35 4815	B9p	≳18 000	Mathys et al. (2023)	3.8	roAp
116114	BD−17 3829	F0p SrCrEu	>17 700	Giarrusso et al. (2022)	6.0	roAp
101065	CD−46 7232	F3p Ho	>15 700 [e]	Hubrig et al. (2018)		roAp
55719	HR 2727	A3p SrCrEu	>14 000	Giarrusso et al. (2022)	6.3	
177765	CD−26 13816	A5p SrEuCr	>13 500	Giarrusso et al. (2022)	3.4	roAp
137949	33 Lib	F0p SrEuCr	≫10 000	Giarrusso et al. (2022)	4.7	roAp
165474	BD+12 3382B	A7p SrCrEu	>9900	Giarrusso et al. (2022)	6.6	
176232	10 Aql	A6p Sr	≫4400	Sikora et al. (2019)		roAp
29578	CD−54 902	A4p SrEuCr	≳4000 [f]	Giarrusso et al. (2022)	3.1	

Notes. Column 1 gives the HD number of the stars (when available) and Col. 2 another identification. The spectral types listed in Col. 3 are from Renson & Manfroid (2009), except when otherwise mentioned in a footnote. Columns 4 and 5 contain the value of the period, when it has been accurately determined, or its lower limit, and the corresponding literature source. For those stars whose spectral lines are resolved into their magnetically split components, the phase-averaged value B_0 of the mean magnetic field modulus $\langle B \rangle$ (the line intensity weighted average over the visible stellar hemisphere of the modulus of the magnetic vector) appears in Col. 6. For stars for which the available $\langle B \rangle$ measurements sample the full rotation cycle well, B_0 is the average of $\langle B \rangle$ over the rotation period; otherwise, it is an average over the observed phases. Most entries in Col. 6 are from Tables 13 and 14 of Mathys (2017); for a few stars, they are complemented by more recent data from Mathys et al. (2019a,b, 2020a, 2023), Giarrusso et al. (2022), and Chojnowski et al. (2023). Rapidly oscillating Ap (roAp) stars are identified in Col. 7. [a]TYC 8979-339-1. [b]Spectral type from Loden et al. (1976). [c]TYC 5395-1139-1. [d]Spectral type from Houk & Cowley (1975). [e]We adopted the time base covered by the $\langle B_z \rangle$ measurements as lower limit, not the heavily extrapolated value of 188 yr proposed by Hubrig et al. (2018). [f]See text.

Among the 44 stars of Table 1, 26 are known to show resolved magnetically split lines. These stars have magnetic fields stronger than the majority of Ap stars. Their predominance among the known ssrAp stars certainly results from a selection bias in part. On the one hand, the most strongly mag-

netic stars are particularly interesting to study to understand better how the magnetic field affects the other physical properties and processes. Thus, they have been the subject of more systematic searches and studies than stars with weaker fields. On the other hand, the consideration of magnetic variations generally

represents the method of choice to constrain the longest rotation periods, because the ratio of their amplitudes to the measurement uncertainties is often much larger than for photometric variations and because long-term instrumental effects can be more easily handled for magnetic measurements than for photometric measurements. These arguments have been developed in greater detail by Mathys (2020).

Nevertheless, one should wonder if the rate of occurrence of super-slow rotation is different in weakly magnetic Ap stars than in strongly magnetic ones. Little information is available in the literature about the magnetic fields of the 18 stars of Table 1 that are not known to show resolved magnetically split lines. Most of them do not appear to have been oberved at a spectral resolution sufficient to derive meaningful constraints about their magnetic fields. The exceptions include HD 8441, for which Titarenko et al. (2012) report that there is no significant magnetic intensification of the spectral line intensities; HD 184471, where the magnetic splitting of the line Fe II λ 6149.2 Å may have been marginally resolved (Mathys 2017); HD 101065 (Przybylski's star), for which a value of 1.6 kG of the mean quadratic magnetic field was derived by Hubrig et al. (2018); and HD 176232, for which several differential line broadening studies yield field strengths (similar to the mean quadratic magnetic field) averaging 1.3 kG (Ryabchikova et al. 2000; Kochukhov et al. 2002; Leone et al. 2003). For a definition of the mean quadratic magnetic field, readers can refer to Mathys & Hubrig (2006); the value of this field moment is generally close to that of the mean magnetic field modulus (see Sect. 6 for details). The mean magnetic field modulus of these four stars must be weaker than 2 kG. Semenko et al. (2011) also noted that the magnetic field of HD 5797 does not exceed 3 kG.

In summary, of the 31 stars of Table 1 for which constraints on the magnetic field are available, 26 have $B_0 \geq 3.0$ kG and five have $B_0 < 3.0$ kG. To which extent this difference is the result of a selection bias that definitely exists or is the reflection of a genuine excess of super-slow rotators among strongly magnetic Ap stars is an important question that needs to be addressed. To this effect, usage of a method that allows ssrAp stars to be identified independently of the strength of their magnetic field is essential. We have proposed an approach that fulfils this requirement. It is based on the argument that Ap stars that do not show photometric variations when observed with TESS (Transiting Exoplanet Survey Satellite) over a 27-d sector are very probably ssrAp stars, except for a few that have $20 \leq P_{rot} \leq 50$ d (still very slowly rotating, but not ssrAp, by definition) and even fewer with an unlikely pole-on geometrical configuration. In previous papers, we applied it to TESS Cycle 1 (Mathys et al. 2020b, hereafter Paper I) and Cycle 2 (Mathys et al. 2022, hereafter Paper II) data. Here, we present its application to analyse TESS Cycle 3 and 4 observations.

Section 2 is devoted to the description of the process of selection of the ssrAp star candidates. Two sources of spurious identifications are discussed in detail: contamination of the original Ap star lists by misclassifications and removal of intermediate period (20 d $\lesssim P \lesssim$ 50 d) variations from the TESS signal as part of its processing. In Sects. 3 and 4, we report the presence of roAp stars and of a δ Sct star among the ssrAp star candidates. The properties of the ssrAp star candidates are discussed in Sect. 5. In Sect. 6, we draw conclusions about the statistical trends revealed by this study and their implications for our understanding of the rotational and magnetic properties of Ap stars.

2. Super slowly rotating Ap (ssrAp) star candidates

2.1. Search strategy

This new step of our project for systematic search of ssrAp star candidates is based on the consideration of the observations obtained during Cycles 3 and 4 of the TESS mission. To select the targets to be analysed, we built a list of the stars whose spectral type contains 'CR', 'EU', 'SR', 'SI' or 'P' in the catalogue of Renson & Manfroid (2009, hereafter the Renson catalogue). This list contains 3110 stars that have a TESS Input Catalog (TIC) identification number. Among the latter, 484 have been observed in Cycle 3 (with 748 corresponding light curves, because for some of the stars, photometric data were obtained in several sectors) and 594 have been observed in Cycle 4 (with 1320 corresponding light curves). We restricted our search to light curves that have 2 min cadence data processed with the TESS SPOC (Science Processing Operations Center; Jenkins et al. 2016) pipeline. This is consistent with the previous papers. We extracted the PDCSAP flux data (for more details, see Sect. 2.4), rejecting datapoints with bad-data flags.

Within the framework of the oblique rotator model, the Ap stars show photometric variability as a result of the non-uniformity of their surface brightness, whose distribution is to first order symmetric about the magnetic axis. The identification of potential ssrAp star candidates from TESS data is based on the consideration that Ap stars that do not show photometric variations over one TESS 27-d sector have a very high probability of having long rotation periods. This assumes that the inclination angle i of the rotation axis to the line of sight and the obliquity angle β between the magnetic and rotation axes are not close to zero. We have shown in detail in Paper I and Paper II that the rate of occurrence of very low i or β is expected to be very low. This discussion will not be repeated here.

Following this approach, we carried out an automated preselection of potential ssrAp stars by calculating on a sector-by-sector basis the Fourier transforms of the 2068 above-mentioned light curves. Those stars showing peaks in the range of $0-10$ d^{-1} with a signal-to-noise ratio (S/N) exceeding 10 were rejected. This left 96 potential ssrAp candidates in Cycle 3 and 151 in Cycle 4.

The areas covered by TESS in Cycles 1 and 3 show considerable overlap. As a result, 42 of the 60 ssrAp star candidates listed in Table 1 of Paper I are among the 96 potential ssrAp star candidates identified in Cycle 3. From this point on, they will be left out of the Cycle 3 list to avoid unnecessary duplication. On the other hand, we checked that for most of the 18 ssrAp star candidates absent from the Cycle 3 list, no 2-min data were obtained in Cycle 3, while the omission of the others can be attributed to low-frequency noise causing spurious peaks with $S/N > 10$ in the $0-10$ d^{-1} frequency range.

Similarly, although the sky coverage of TESS Cycle 4 differs considerably from that of Cycle 2, 28 of the 151 stars returned by the automatic search procedure for Cycle 4 were already listed in Table A.1 of Paper II, and 1 was present in Table 1 of Paper I. These stars were accordingly removed from the Cycle 4 list. After completion of these steps, there were 54 stars left in the Cycle 3 list and 122 stars left in the Cycle 4 list. Two stars, TIC 152803574 and TIC 372617495, appear in both lists (see also Appendix A and Sect. 2.3).

2.2. Classification issues

The initial automatic identification of the stars from the Renson catalogue for which TESS 2-min observations were obtained in

Cycles 3 and 4 needs to be refined for a number of reasons. In particular, the resulting lists include stars flagged with either '/' or '?' in the first column of the catalogue. Renson & Manfroid (2009) assigned the '/' flag to stars that were once improperly considered as chemically peculiar. The Cycle 3 and 4 lists contained, respectively, two and three such stars. Making use of the classification and literature information available through SIMBAD, we confirmed that none of these five stars is indeed an Ap star.

The flag '?' in the Renson catalogue identifies stars whose peculiar nature is doubtful. Consistently with our approach of Paper II, to ensure that the final list of ssrAp star candidates is restricted as much as possible to bona fide Ap stars, we excluded the stars flagged '?' by Renson & Manfroid (2009) unless evidence of their peculiar nature was subsequently brought forward, according to the information available through SIMBAD. Such evidence was found only for one of the 13 stars flagged '?' from the Cycle 3 list, the roAp star TIC 349945078 (Balona et al. 2019). No convincing evidence that any of the 31 flag '?' stars of the Cycle 4 list is a bona fide Ap star was found.

After exclusion of the stars flagged '/' or '?' (except for TIC 349945078), 40 entries were left in the Cycle 3 list, and 88 in the Cycle 4 list. Some of these are affected by another ambiguity that is apparent in the Renson catalogue. Namely, the spectral type reported for some stars both contains a hyphen ('–'), which distinguishes Am stars, and lists an overabundant chemical element (e.g., Sr), which is the attribute of an Ap star. Of the six stars of this type from the Cycle 4 list, five are definitely Am stars, and one, TIC 2934856, is an Ap star (Shi et al. 2023). The five Cycle 3 stars with similar classification information are all Am or normal A stars.

Moreover, for some stars, the notes to the Renson catalogue indicate that the validity of the classification appearing in the main table is questionable – most often, there exists an ambiguity between Ap and Am classifications. We reviewed these cases individually. Only TIC 248354858 appears to be an Am star; it was excluded from the Cycle 3 list.

After the exclusion of the Am and normal A stars discussed in the two paragraphs above, the Cycles 3 and 4 candidate lists were reduced to 34 and 83 stars, respectively. As the next step, information available in the literature (as listed in SIMBAD) for the remaining stars was reviewed. Particular attention was paid to spectral classification and abundance analyses, period and $v \sin i$ determinations, and magnetic field determinations.

Among the remaining 34 stars of the Cycle 3 list, we found four that have been definitely confirmed to have peculiarities that are different from those of magnetic Ap stars: one Am star, one HgMn star, one F str $\lambda 4077$ star, and one Ba star. Furthermore, there is one normal late B star that was classified as such in a study specifically devoted to the identification of Ap stars. Also, in his extensive, systematic study of the stellar magnetic fields, Babcock (1958) reported a lack of lines, of which there are only traces, in the spectrum of HD 45827. A high-resolution spectrum ($R \simeq 70\,000$) from our collection, recorded with the AURE-LIE spectrograph at the Observatoire de Haute-Provence (OHP), does not show any lines either. Accordingly, HD 45827 appears very unlikely to be an Ap star. Finally, for one star, the only references listed in SIMBAD are the two editions of the Renson catalogue (Renson et al. 1991; Renson & Manfroid 2009) and no MK spectral type is given. The source of the Ap classification reported in the Renson catalogue is unknown. This star cannot be considered a bona fide Ap star.

The arguments presented in the previous paragraph lead us to exclude the seven stars to which they refer from the list of

the Cycle 3 ssrAp candidates, on account that either they are definitely not Ap stars, or their Ap nature is not convincingly established. Thus, the number of entries in the list is reduced to 27.

A more severe issue was identified when carrying out a similar literature review for the Cycle 4 list of candidate ssrAp stars. For 51 of these 83 stars, the only source that we could identify for the Ap classification reported by Renson & Manfroid (2009) is the catalogue of spectral classifications of stars in an area of 140 square degrees in the Galactic anticentre direction that was compiled by the Abastumani Astrophysical Observatory (Chargeishvili 1988). The 208 Ap stars discovered as part of the study performed to build this catalogue were listed by Kharadze & Chargeishvili (1990).

The automatic search procedure that we applied found that 51 of these 208 stars, or 25%, do not show photometric variations at low frequencies typical of Ap star rotation. Taken at face value, this suggests that the rate of occurrence of super slow rotation among Ap stars in the direction of the Galactic anticentre is much higher than that rate among Ap stars in general, which does not considerably exceed 10% (Mathys 2017, 2020; Netopil et al. 2017; Shultz et al. 2018; Labadie-Bartz et al. 2023). That the rate of occurrence of super slow rotation is much higher than the average value in a limited region of the sky such as a patch around the Galactic anticentre appears implausible. It is much more likely that a considerable fraction (more than half) of the stars classified as Ap in the Abastumani catalogue are actually normal A (or Am) stars. This is all the more probable since all of those stars for which an MK spectral type was available in SIMBAD (mostly from the Michigan Spectral Survey) were classified as normal A stars. Thus, while some of the stars from the list of Kharadze & Chargeishvili (1990) must almost certainly be genuine Ap stars, none of them can be regarded as a bona fide Ap star unless there exists some other convincing evidence of its peculiarity. Such evidence was only found for two of the 51 Abastumani stars that we identified as candidate ssrAp stars through the above-described automatic analysis of the TESS data. For these two stars, short photometric periods possibly attributable to rotation were reported in the literature, so that they may not be super slowly rotating anyway. This will be further discussed below. The other 49 stars for which the only identified Ap classification source is the Abastumani catalogue were removed from the Cycle 4 list of ssrAp star candidates, leaving only 34 entries in this list.

Based on the information available in the literature, nine more of these 34 stars were not considered as bona fide magnetic Ap stars. Five have unambiguous, different classifications: two are F str $\lambda 4077$ stars, one is a HgMn star, one is an Am star and one is a normal F0 star. One more is an ambiguous case: depending on the source of the classification, it may either be an Ap Si star or an Am star. In two cases, we could not identify the source of the peculiar classification; only one has an MK spectral type reported in SIMBAD, as a normal A star. For the ninth one, the abundances derived from the analysis of spectra recorded at a spectral reasolution $R \simeq 34\,000$ are mostly normal. These nine stars were also excluded from the Cycle 4 ssrAp candidate list. One of them, the F str $\lambda 4077$ star TIC 152803574, was also observed in Cycle 3, and removed from the Cycle 3 list of the ssrAp star candidates on account of misclassification.

In summary, a total of 64 stars that were assigned an Ap spectral type in the Renson catalogue without any indication of classification uncertainty were removed from our selection of ssrAp star candidates because, after critical evaluation, we do

not regard them as bona fide Ap stars. These stars are listed in Tables A.1 and A.2.

2.3. Visual inspection of the amplitude spectra

For the remaining 27 stars of the Cycle 3 list and 25 stars of the Cycle 4 list, we proceeded to the visual inspection of the amplitude spectra to confirm the lack of low-frequency variability that led to their selection by the automatic search procedure. Five of the Cycle 3 stars and 13 of the Cycle 4 stars proved to show rotational variability with frequencies $f > 0.049\,\mathrm{d}^{-1}$. The latter include the two above-mentioned Ap stars from the Abastumani catalogue for which we regarded the peculiar nature as confirmed since short periods had been reported in the literature. However, only for one of them does the value of the published period coincide with the one derived from analysis of the TESS data.

The exclusion of the stars for which super slow rotation was ruled out through visual inspection of the amplitude spectra left 22 and 12 ssrAp candidates in the Cycle 3 and Cycle 4 lists, respectively. However, the star TIC 372617495 (=HD 48953) appears in both lists. Accordingly, we are left with 33 stars that, after critical evaluation, we regard as bona fide Ap stars, for which the above-described analysis of the TESS data does not show any low-frequency variability. In Papers I and II, we concluded that Ap stars selected in that way must have rotation periods considerably longer than one TESS 27-d sector, except for the infrequent occurrence of very small inclination angle i or obliquity angle β. Thus, it appeared that almost all of these stars must be ssrAp stars, according to the definition of Mathys (2020), that is, having $P_{\mathrm{rot}} > 50\,\mathrm{d}$. In the next section, we shall show that, in fact, variations with moderately long periods ($20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$) may have escaped detection up to this point.

2.4. Long period stars with measurable rotation periods $20 \lesssim P_{\mathrm{rot}} \lesssim 50\,d$

TESS data are delivered in two forms, the simple aperture photometry (SAP) and pre-search data conditioning SAP (PDC-SAP). The SAP suffers from instrumental drifts and zero point shifts that are modelled with ensembles of stars on the same CCD to produce the co-trending basis vectors (CBV) that are then used to remove these instrumental effects sector-by-sector. The primary purpose is to optimise the data to search for exoplanet transit signals. One consequence of the application of the CBV is that long-term astrophysical trends can also be removed in the PDCSAP data. Claytor et al. (2024) discuss this in detail in a study of rotation signals in TESS data for cool stars.

For stars with rotation periods of the order of, or greater than, the TESS sector length (27 d), some have amplitudes greater than the instrumental variations at low frequencies so that we can determine the rotational period despite the instrumental noise. Because of the many kinds of instrumental variations that occur, caused by, for example, telescope operations, changing background, changing contamination, and data download gaps, we are cautious in determining periods from the SAP data. Our purpose in this work is to find ssrAp stars with rotation periods longer than 50 d for spectroscopic follow-up to study the magnetic fields. In this section we only show cases of Ap stars that have rotation periods that can be determined from the SAP data. There are 9 such stars, 8 of which have periods in the range $20 \lesssim P_{\mathrm{rot}} < 50\,\mathrm{d}$, and one of which is a bona fide ssrAp star with $P_{\mathrm{rot}} \gtrsim 80\,\mathrm{d}$.

Our technique for searching for ssrAp stars, as described in the previous sections, is to find Ap stars that show no low-frequency variations in the PDCSAP data. Stars with $P_{\mathrm{rot}} \lesssim 20\,\mathrm{d}$ do not pass this test, as the signal from the rotation is generally seen. However, there is a grey area for $20 \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$ where the CBV remove much or all of the rotation signal in the PDC-SAP data, but the signal can still be seen in the SAP data. These stars are not ssrAp stars, but they are still of interest, hence we note them here.

Their rotation periods are indeed longer than those of the vast majority (about 90% – see for instance Fig. 1 of Netopil et al. 2017 and Fig. 4 of Labadie-Bartz et al. 2023) of Ap stars. They have lost a considerable amount of angular momentum and their consideration as an intermediate case between the most numerous stars with rotation periods of a few days and the extreme ssrAp stars can be expected to provide additional insight into the braking mechanisms that have been (and possibly are still) at play. Furthermore, their projected equatorial velocity is at most of the order of $7\,\mathrm{km\,s^{-1}}$, so that their most magnetically sensitive spectral lines, such as Fe II $\lambda\,6149.2\,\text{Å}$, can be partly resolved into their magnetically split components for magnetic fields with a mean modulus of the order of $3\,\mathrm{kG}$. Such fields are weak enough to be of relevance to constrain the possible connection between magnetic field strength and the rate of occurrence of (super) slow rotation in Ap stars (see Paper II and Sect. 6).

2.4.1. A comparison of the SAP and PDCSAP data for an example star: TIC 162027140 (HD 97394)

We illustrate here the impact of the CBV on long-term rotational variations in the stars that show variability in the SAP data but not in the PDCSAP data with the example of TIC 162027140 (HD 97394). This star shows remarkable overabundances of some rare earth ions up to a million times solar and has resolved magnetically split lines, seen on UVES spectra (Elkin et al. 2011), that clearly indicate a long rotation period. This star is an ssrAp star candidate.

Yet the PDCSAP data are problematic. This can be seen in Fig. 1 where a $47\overset{d}{.}38$ rotational variation is apparent in the SAP data, but is removed by the pipeline application of the CBV in the PDCSAP data. (In the SAP data we adjusted the zero point of S36 to match that of S37.) The star has a double wave with the secondary maximum being almost flat. If we assume that there are two major spots near the magnetic poles, with the magnetic axis inclined to the rotation axis (as is typical of α^2 CVn Ap stars), then one pole crosses near to the line of sight, and the other skims the observed limb. We have not modelled this, but speculatively it indicates a geometry with the rotational inclination, and magnetic obliquity $i, \beta \sim 45°$. Future long-term magnetic measurements will clarify this.

2.4.2. Determination of P_{rot} from the SAP data for slowly rotating Ap stars

We show here the SAP light curves and low-frequency amplitude spectra for stars where a measurement, or estimate, of $P_{\mathrm{rot}} \gtrsim 20\,\mathrm{d}$ is possible. For the stars with periods in the range $20-30\,\mathrm{d}$ and one 27-d sector of data, only one cycle, or less, has been observed, making the period determined somewhat uncertain. Also, α^2 CVn stars often show a double wave variation as a consequence of spots at both magnetic poles, hence the periods derived from one cycle of data may be only half the true period. For stars that have more than one sector of data, in some cases a zero point correction has been applied to particular sectors.
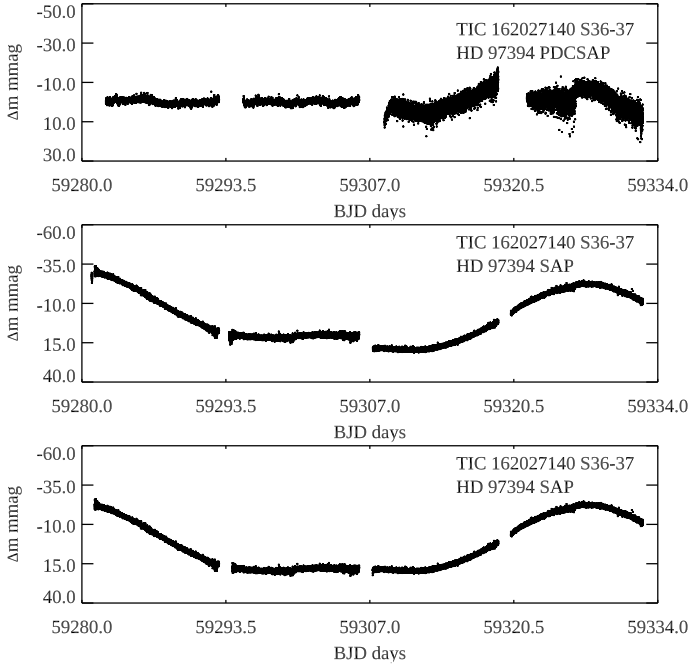
Fig. 1. TIC 162027140 (HD 97394). Top panel: light curve of the S36-37 PDCSAP data where the removal of long-term trends by the CBV has generated low-frequency problems. Middle panel: SAP data where the 47$^{\rm d}$38 rotational variation can be seen. There is also an instrumental zero-point shift between S36 and S37. Bottom panel: SAP data with adjusted zero-point shift.

Obvious outliers have been trimmed, along with some short sections of data on the edges of gaps where there are data reduction problems. Figures 2 and 3 show the SAP light curves for nine stars in our sample where the rotation period was determined, or estimated.

We note that Labadie-Bartz et al. (2023) have recently looked at rotation periods for stars classified as magnetic Ap stars with LAMOST spectra. These authors used the Full-Frame Images (FFI) up to Sector 35. For S1-26 the integration time was 30 min, then for S27-35 it was 10 min. Hence Labadie-Bartz et al. (2023) do not use the 120-s cadence we used, so are not sensitive to any possible roAp pulsations. There is overlap between their study and ours of only one star, TIC 2934856 (HD 281056), where we agree that $P_{\rm rot} = 34$ d.

2.4.3. Notes for Figs. 2 and 3

TIC 2934856 (HD 281056): Labadie-Bartz et al. (2023) found $P_{\rm rot} = 34^{\rm d}35$; we find $P_{\rm rot} = 34^{\rm d}0$ in this work. These are in agreement within the uncertainties.

TIC 96855460 (HD 185256) is a known roAp star; see Sect. 3. With the value that we derive for the rotation period, $P_{\rm rot} = 25^{\rm d}64$, this is not an ssrAp star.

TIC 162027140 (HD 97394) has $P_{\rm rot} = 47^{\rm d}38$, close to our lower limit of 50 d for ssrAp stars.

TIC 276300910 (HD 134799A) is not resolved from its visual companion HD 134799B in the TESS data. The SAP data show a rotation signal with $P_{\rm rot} = 22^{\rm d}6$ in both the S11 and S38 data, separately, and $P_{\rm rot} = 23^{\rm d}0$ for the combined sectors. Because there is only one cycle per sector, and there is a large gap between S11 and S38, this value is not inconsistent with that reported by Netopil et al. (2017), $P_{\rm rot} = 25^{\rm d}373$. The period is

clearly moderately long, but TIC 276300910 is not an ssrAp star according to our definition.

TIC 286255541 (HD 69913) has data from S8-9, S34-36, and S61-63. All phase well with the determined period $P_{\rm rot} = 19^{\rm d}98$. This is in good agreement with the value $P_{\rm rot} = 19^{\rm d}975$ derived by Bernhard et al. (2015a) from the analysis of ASAS-3 photometric data.

TIC 312839768 (HD 125525) has a derived period $P_{\rm rot} = 26^{\rm d}70$ equal to the length of the S38 data, also $26^{\rm d}70$, and the amplitude is relatively low. The similarity of this value with the one that Hümmerich et al. (2016) determined using data from the ASAS-3 archive suggests that the variations seen in the SAP light curves is of stellar origin rather than due to an instrumental effect.

TIC 405516045 (HD 110274) shows a long-term drift. This star, whose spectrum shows resolved magnetically split lines (Freyhammer et al. 2008), was already a known ssrAp star. Its rotation period, $P_{\rm rot} = 265^{\rm d}3$, was determined by Freyhammer et al. (2008) from the analysis of ASAS-3 photometric data. While this period is much longer than the time span covered by the available TESS observations, the variation trend is clearly seen in the SAP data, which suggests $P_{\rm rot} \gtrsim 80$ d.

TIC 427032468 (HD 140748) potentially has a double-wave light curve, in which case $P_{\rm rot} = 50^{\rm d}8$ and this is a ssrAp star. However, this is uncertain, as the period value, $P_{\rm rot} = 25^{\rm d}4$, differs considerably from, but is not inconsistent with that published by Netopil et al. (2017), $P_{\rm rot} = 36^{\rm d}520$. Because the TESS data do not cover full cycles for these periods, the data can be reasonably phased with all three periods, hence do not distinguish amongst them.

TIC 429553678 (HD 40678): the value of the period that we estimate, $P_{\rm rot} = 26^{\rm d}9$, and the value reported by Hümmerich et al. (2016), $P_{\rm rot} = 22^{\rm d}029$, based on the analysis of photometric data from the ASAS-3 archive, are not incompatible.

For all of the stars in this subsection, it is difficult to determine precise rotation periods from one cycle, or less, of light curves that are generally not sinusoidal. The values we provide are guides to future studies, particularly of the magnetic fields.

2.5. TESS Cycles 3 and 4 ssrAp candidates

In summary, of the nine stars whose SAP light curves are discussed in Sect. 2.4.3, only TIC 405516045 is a (definite) ssrAp star. Although TIC 162027140 and TIC 427032468 are borderline cases, we exclude the remaining eight stars of this section from the final list of ssrAp star candidates identified as part of our systematic search based on the TESS Cycles 3 and 4 photometric data. Thus, this final list comprises 25 stars, for which the relevant data are presented in Table 2. The amplitude spectra for the stars in this table are shown in Appendix B. We also present in Table 3 similar data for the eight stars with moderately long rotation periods from Sect. 2.4.3, given their relevance in the context of the understanding of the differentiation of rotation in Ap stars (as argued in Sect. 2.4).

3. The roAp stars

Four of the 25 ssrAp stars announced in this paper, and one of the stars with 20 d $\lesssim P_{\rm rot} \lesssim$ 50 d are also roAp stars. Thus, there are four roAp stars among the 25 ssrAp stars listed in Table 2, making up 16% of the sample. In Papers I and II, we found 22%
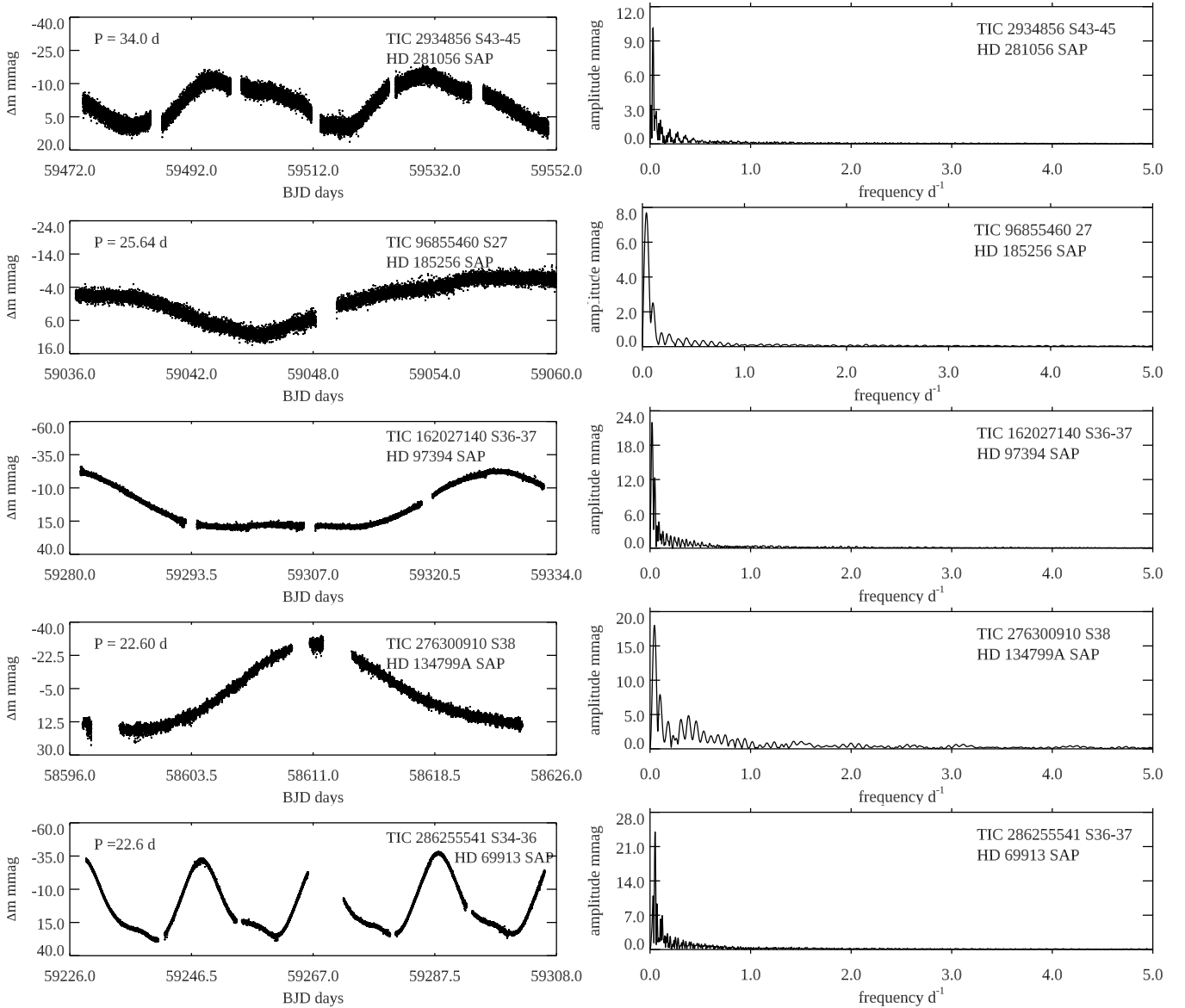
Fig. 2. SAP light curves and low-frequency amplitude spectra for Ap stars with measurable periods $P_{rot} \gtrsim 20$ d. Some periods in the range $20-30$ d for those stars with only one sector are uncertain because of the non-sinusoidal shape of α^2 CVn light curves.

and 19% in the southern (Cycle 1) and northern (Cycle 2) TESS data, respectively. These fractions are similar and are significantly greater than the 5.5% overall occurrence rate for roAp stars among all Ap stars studied in TESS data (Holdsworth et al. 2021, 2024). Thus, there is a positive correlation with the occurrence of roAp pulsation and slow rotation, even though some roAp stars have rotation periods as short as 2 d.

We discuss the five roAp stars in this work individually below. Their amplitude spectra are shown in Fig. 4.

3.1. TIC 36576010 (HD 216018)

TIC 36576010 (HD 216018) was discovered to be an roAp star by Balona (2022). There are two significant peaks seen in the amplitude spectrum at 155.464 d^{-1} and 166.686 d^{-1} with amplitudes of 32 ± 6 µmag and 45 ± 6 µmag, respectively. These differ in frequency by 130 µHz, which is possibly twice the large separation. With these low amplitudes and low S/Ns, pulsations in this star will be difficult to study further.

3.2. TIC 96855460 (HD 185256)

TIC 96855460 (HD 185256) is a known roAp star. Kurtz & Martinez (1995) reported a single pulsation mode with a frequency of 140.8 d^{-1}. The TESS S27 data show a single peak at 140.6 d^{-1}. The pulsation was also detected in WASP data with a frequency of 141.18 d^{-1} (Holdsworth et al. 2014). This star is not a ssrAp star, as seen in Fig. 2.

3.3. TIC 189996908 (HD 75445)

TIC 189996908 (HD 75445) is a known roAp star with an exceedingly low pulsation amplitude of $20-40$ m s^{-1} in radial velocity measurements of Nd II and Nd III lines (Kochukhov et al. 2009). Those authors found multiple pulsation frequencies in the $1.8-2.0$ mHz range ($155-173$ d^{-1}). We find two 6σ peaks at 159 d^{-1} and 170 d^{-1}, consistent with the spectroscopic results. Kochukhov et al. (2009) also find a marginal rotational variation with a period of $P_{rot} = 29\overset{d}{.}5$ in ASAS data. We do not see this period in the S8-9, S35 SAP
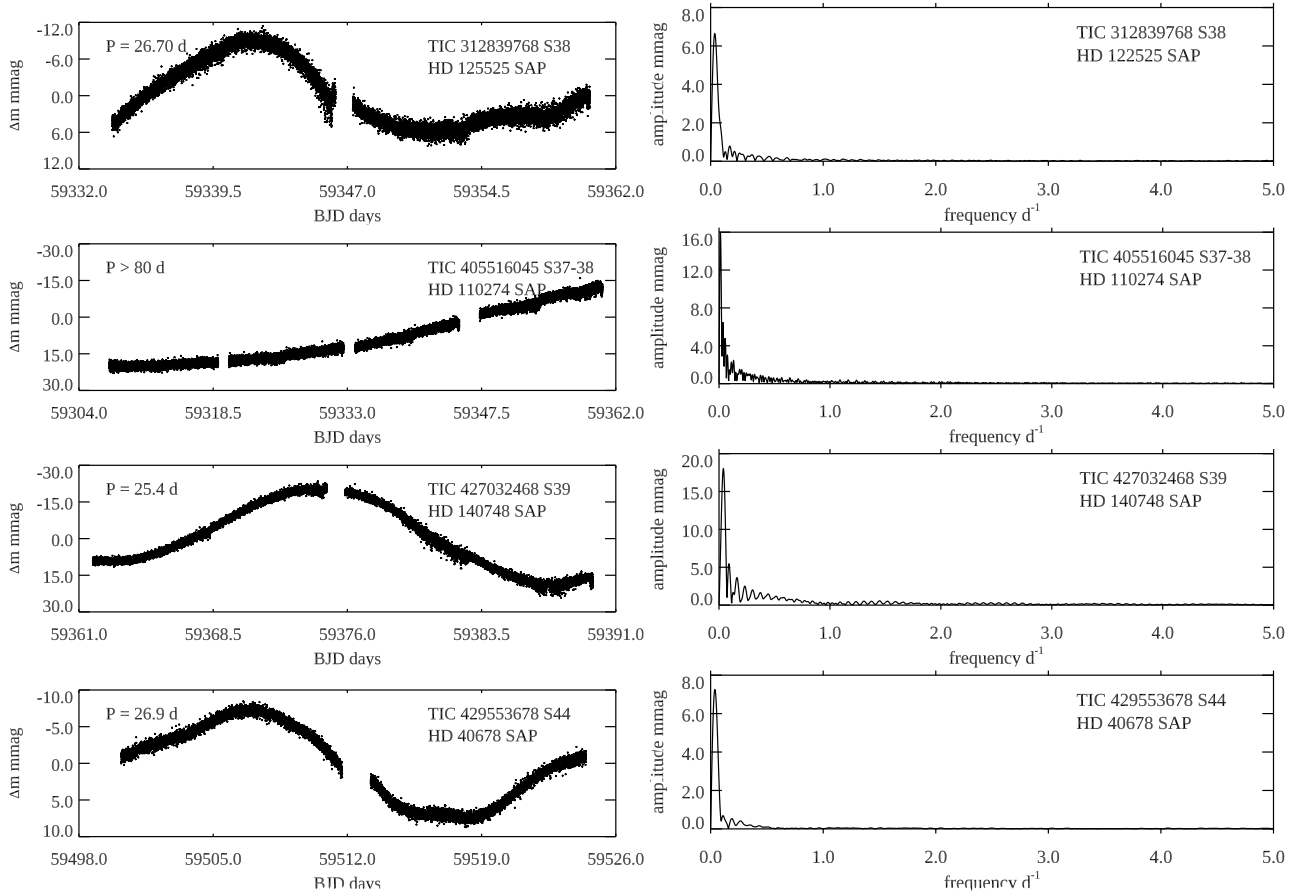
Fig. 3. SAP light curves and low-frequency amplitude spectra for Ap stars with measurable periods $P_{\rm rot} \gtrsim 20$ d – continued.

data. From the consideration of mean magnetic field modulus measurements, Giarrusso et al. (2022) suggest that the rotation period must be much longer than 14 yr, if the star shows any variability.

3.4. TIC 299000970 (HD 176232; 10 Aql)

TIC 299000970 (HD 176232; 10 Aql) is a known roAp star. Heller & Kramer (1990) discovered rapid oscillations in 10 Aql and identified three pulsation frequencies at 107, 120, and 124 d^{-1}. Kochukhov et al. (2002) found low amplitude (30 m s^{-1}) radial velocity pulsational variations in a spectroscopic study of the Nd III λ 6145 Å line at frequencies similar to those of Heller & Kramer. The amplitude spectrum for 10 Aql in Fig. 4 shows multiple pulsation frequencies. The four highest amplitude peaks are at 115.4934 d^{-1}, 120.6831 d^{-1}, 123.2995 d^{-1}, and 125.0940 d^{-1} with the uncertainty in the final digit. These are in the same range as found by Heller & Kramer. The amplitudes are mostly stable over S53-54 and there is no indication of rotational modulation. The separations in frequency for those four peaks are in the 20−50 µHz range, which is plausibly representative of the large separation and half of that, as expected for dipole and quadrupole modes that are typical for roAp stars.

3.5. TIC 354619745 (HD 201601; γ Equ)

TIC 354619745 (=γ Equ) is a famous ssrAp star with a rotation period of about 1 century (Bychkov et al. 2016). It is also

a known roAp star that has been studied photometrically for 40 years.

Kurtz (1983) discovered pulsation in γ Equ from 11 nights of ground-based photometry in 1981. He detected only one pulsation frequency at 115.7 d^{-1}. In a multi-site campaign, Martinez et al. (1996) obtained 26 nights of ground-based photometry from six observatories over a time span of 40 d in 1992. They found four pulsation frequencies, 115.69 d^{-1}, 118.02 d^{-1}, 120.70 d^{-1}, and 123.29 d^{-1}. They noted the frequency spacing is about 30 µHz, which they interpreted as half the large separation, implying alternating even and odd degree modes. That led to an asteroseismic luminosity measurement in good agreement with the astrometric luminosity.

A comprehensive study was performed on 19 d of data obtained with the MOST space mission (Gruberbauer et al. 2008). Those authors give a history of previous photometric and radial velocity studies of pulsation in this star. They found pulsation mode frequencies of 117.90 d^{-1}, 117.97 d^{-1}, 123.30 d^{-1}, 120.00 d^{-1}, and 113.26 d^{-1}, in order of decreasing amplitude, along with harmonics of the first and third frequencies. We note that their first two frequencies are not fully resolved in their data set, suggesting amplitude modulation.

We find three certain significant mode frequencies in the S55 TESS data, as seen in Fig. 4. They are $\nu_1 = 115.6564$ d^{-1}, $\nu_2 = 118.0240$ d^{-1}, and $\nu_3 = 113.2895$ d^{-1}, also in order of decreasing amplitude. The uncertainties are in the final digit. We also see the harmonic $2\nu_1$ and the combination term $\nu_1 + \nu_2$. The three frequencies are not quite equally spaced with separations

Table 2. ssrAp star candidates found by our technique in the TESS Cycles 3 and 4 data.

TIC	HD	Spectral type	V (mag)	$T_{\rm eff}$ [a] (K)	$\log g$ [a] (cm s^{-2})	roAp	$\langle B_z\rangle_{\rm rms}/B_0/Q_0$ (kG)	Refs.	$P_{\rm rot}$ (d)	Refs.	Lines [b] $v \sin i$ (km s^{-1})	Refs. [c]	Notes	TESS sectors
32259138	138777	A3p SrEu	9.73	7350	3.90		2.1/–/–	1			25	1		51
36576010	216018	A7p SrEuCr	7.62	7750	4.05	roAp	1.4/5.6/6.8	2, 3	34.044 [d]	2	r	4		42
49159482	179246	B9p EuCr	9.82	8320										54
88202438	192686	A0p Si	8.88	11670	3.75									54
126975139	126297	A5p CrEuSr	9.49	7640	3.70		–/–/1.0	5			4	5		38
169382402	32996	A0p Si	6.04	10420	3.95						15	6		32
189996908	75445	A3p SrEu	7.12	7610	4.00	roAp	0.1/3.0/4.3	3	>5000?	2	r	4		35
209708422	122379	A0p Si	10.45	13120										38
253260234	113149	A0p CrEu	10.14	10310	4.15									37–38
262956098	3988	A0pCrEuSr	8.35	8470	3.60		–/2.65/–	3			r	7	SB2 (7)	39
276354649	134874	B9p Si	7.67	12070							30	8		38
282468249	197077	A2p SiSr	9.38	8177	4.15									55
295698744	119933	A2p SrCrEu	9.27	6640	3.90									50
299000970	176232	A6p Sr	5.89	7520	3.85	roAp	0.4/–/1.3	9, 10–12	>12 yr	9	2	11		53-54
334327860	111675	A0p EuSr	9.74	8620	3.80									38
354619745	201601	A9p SrEu	4.68	7490	4.05	roAp	1.0/3.9/5.2	3	>97 yr	13	r	14		55
369969602	128472	A2p CrSrEu	9.87	7260										38
372617495	48953	Fp SrEu	6.80	6910	3.70				2.8939 [e]	15				44–45
380607580	119794	A2p CrEuSr	9.00	9350	3.95									38
405516045	110274	A0p EuCr	9.47	7770	3.85		–/4.0/–	3	265.3	16	r	16		37–38
405557056	155102	A2p Si	6.36	9100	3.80						38	17		51–53
410451752	66318	A0p EuCrSr	9.56	10130	4.10		6.6/14.5/–	3,18,19			r	18		34–37
425796196	138146	A0p EuCr	10.07	7980	3.95									39
438694338	117227	A0p CrSr	9.12	7800	4.35						b	F		38
442695956	114568	A0p Si	10.18	12390										38

Notes. The stars are ordered by increasing TIC number, as given in Col. 1. The HD numbers are listed in Col. 2, as alternative identifications. The spectral types appearing in Col. 3 are from the Renson catalogue; the V magnitudes in Col. 4 were extracted from SIMBAD; the effective temperatures $T_{\rm eff}$ and surface gravities $\log g$ in Cols. 5 and 6 were taken from the TIC. The roAp stars (see Sect. 3) are identified in Col. 7. Column 8 presents the magnetic data available in the literature, with the relevant references in Col. 9. The values of up to three magnetic moments are given; all of them correspond to the mean of the respective moment over a rotation period (if known and adequately sampled by the existing measurements) or over the observations that have been obtained (otherwise). The first one is the root-mean square longitudinal magnetic field, $\langle B_z\rangle_{\rm rms}$, as defined by Bohlender et al. (1993). In essence, this is the quadratic mean of the mean longitudinal magnetic field (the line-intensity weighted average over the stellar disc of the component of the magnetic vector along the line of sight). The second one is the average value B_0 over a rotation cycle of the mean magnetic field modulus $\langle B\rangle$ (that is, the same quantity as listed in Col. 3 of Table 13 of Mathys 2017). The third field moment, Q_0, is the average value over a rotation period of the mean quadratic magnetic field $\langle B_q\rangle$; this is the same quantity as appearing in Col. 10 of Table 13 of Mathys (2017). The period values from the literature appear in Col. 10. They come from the references specified in Col. 11. Column 12 contains the $v \sin i$ values (as numbers), when they could be found, or indications about the resolution or width of the spectral lines (as letters). The references from which this information originates appear in Col. 13. For some stars, a note was added in Col. 14. The last column indicates in which TESS 27-d sectors the data analysed in this study were obtained. [a] Values retrieved from the TIC where available. [b] r = resolved; b = broad. [c] F: the line width information is based on visual inspection of a FEROS spectrum of our collection. [d] This value of the period seems spurious. The star more likely has a rotation period vastly exceeding 6 yr, as first suggested by Mathys (2017). See text for details. [e] No low-frequency is seen in the TESS data. See text for details.

References. (1) Romanyuk et al. (2017); (2) Giarrusso et al. (2022); (3) Mathys (2017); (4) Mathys et al. (1997); (5) Ryabchikova & Romanovskaya (2017); (6) Abt & Morrell (1995); (7) Elkin et al. (2012); (8) Levato et al. (1996); (9) Sikora et al. (2019); (10) Ryabchikova et al. (2000); (11) Kochukhov et al. (2002); (12) Leone et al. (2003); (13) Bychkov et al. (2016); (14) Scholz (1979); (15) Wraight et al. (2012); (16) Freyhammer et al. (2008); (17) Zorec & Royer (2012); (18) Bagnulo et al. (2003); (19) Bagnulo et al. (2006).

of 27.4 μHz. This is half the large separation, so the modes are probably alternating $\ell = 2, 1, 2$, assuming the mode with highest amplitude is a dipole. This is consistent with the result of Martinez et al. (1996).

After pre-whitening the three mode frequencies the amplitude spectrum of the residuals shows very little (Fig. 4). There is a small indication of mild amplitude modulation of ν_1. The highest amplitude peak at $\nu_1 = 115.6564\,{\rm d}^{-1}$ is in agreement with those found by Kurtz (1983) and Martinez et al. (1996), but this peak is not found at all in the MOST data of Gruberbauer et al. (2008), although ν_2 and ν_3 are. Previous studies have suggested that amplitude modulation of the modes occurs on time scales as short as 1 d (see Gruberbauer et al. 2008 for a discussion). This is plausible, since strong amplitude modulation on a time-scale of

only days is known in other roAp stars, most notably HD 60435 (Matthews et al. 1987; Balona et al. 2019).

4. A δ Sct star

TIC 276300910 (HD 134799) shows clear rotation with $P_{\rm rot} = 22^{\rm d}6$. Figure 5 shows the amplitude spectra for this star, where clear δ Sct pulsations can be seen. δ Sct pulsations are rare in magnetic Ap stars and none is known to pulsate with rich, multiple modes as seen in Fig. 5. Those probably arise in TIC 276300912 (HD 134799B), which is an A5 star in the δ Sct instability strip that is unresolved from TIC 276300910 in the TESS photometry.

Table 3. Stars with moderately long rotation periods ($20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$) found in the TESS Cycles 3 and 4 data.

TIC	HD	Spectral type	V (mag)	T_{eff} [a] (K)	$\log g$ [a] (cm s^{-2})	roAp	$\langle B_z \rangle_{\mathrm{rms}}/B_0/Q_0$ (kG)	Refs.	P_{rot} (d)	P_{litt} (d)	Refs.	Lines [b] $v \sin i$ (km s^{-1})	Refs. [c]	Notes	TESS sectors
2934856	281056	A5-	10.68	8910	3.45				34.0	34.3	1				43–45
96855460	185256	F0p SrEu	9.96	7100	4.00	roAp	0.7/–/1.4	2, 3	25.64			5.5	3		27
162027140	97394	A5p EuCrSr	8.76	8360	3.75		–/3.1/–	4	47.38			r	4		36–37
276300910	134799A	A7p SrCrMg	7.94	7840	4.25				22.6	25.373	1			δ Sct [d]	38
286255541	69913	B9p Si	8.20	9880	3.50				19.98	19.975	5				34–36
312839768	122525	A0p EuSrCr	8.71	8330	3.55		–/–/1.7	6	26.70	26.072	7	7	6		38
427032468	140748	A2p EuCr	9.22	8490	3.80				50.8	36.52	1	vs	F		39
429553678	40678	A0p SiSr	7.38	9520	3.70				26.9	22.029	7				44

Notes. Columns 1–9 are the same in Table 2, in which Col. 10 has no counterpart. It contains the values of the moderately long periods that were determined in the course of this study (see Sect. 2.4.2). The data presented in Cols. 11–16 of are of the same nature as those given in Cols. 10–15 of Table 2. [a] Values retrieved from the TIC where available. [b] r = resolved; vs = very sharp. [c] F: the line width information is based on visual inspection of a FEROS spectrum of our collection. [d] The δ Sct pulsation signal probably originates from the companion, HD 134799B. See text for details.
References. (1) Netopil et al. (2017); (2) Hubrig et al. (2004); (3) Kochukhov et al. (2013); (4) Elkin et al. (2011); (5) Bernhard et al. (2015a); (6) Ryabchikova & Romanovskaya (2017); (7) Hümmerich et al. (2016).
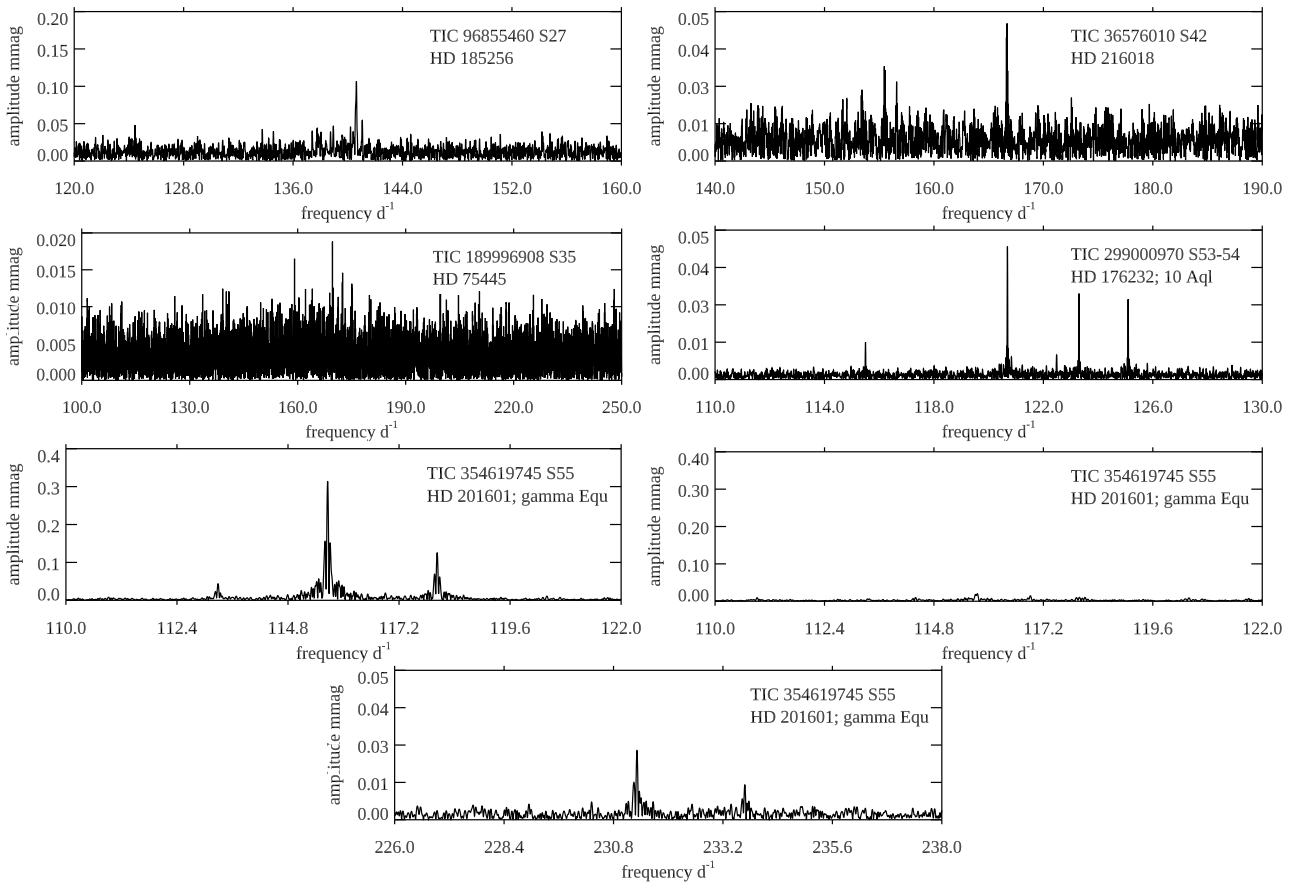

Fig. 4. Amplitude spectra for 5 roAp stars discussed in the text. The bottom three panels are for γ Equ, showing the three mode frequencies, the amplitude spectrum of the residuals after pre-whitening those three frequencies, and the $2\nu_1$ harmonic and $\nu_1 + \nu_2$ combination term.

5. Discussion

5.1. The ssrAp star candidates

Three of the ssrAp star candidates of Table 2 were already known ssrAp stars, listed in Table 1: HD 110274 ($P_{\mathrm{rot}} = 265^{\mathrm{d}}3$), HD 176232 ($P_{\mathrm{rot}} > 12\,\mathrm{yr}$) and HD 201601 ($P_{\mathrm{rot}} > 97\,\mathrm{yr}$). (The references for the periods are given in the tables.) In the spectra of the former two, the lines are resolved into their magnetically

split components. The lines of HD 176232 are very sharp but it was still possible to constrain the magnetic field strength by analysing the differential broadening of lines of different magnetic sensitivities (Ryabchikova et al. 2000; Kochukhov et al. 2002; Leone et al. 2003). Furthermore, if HD 75445 (which is also identifed as a ssrAp star candidate in Table 2 and also shows resolved magnetically split lines) is variable at all, its period must be considerably longer than 14 years (Giarrusso et al. 2022).
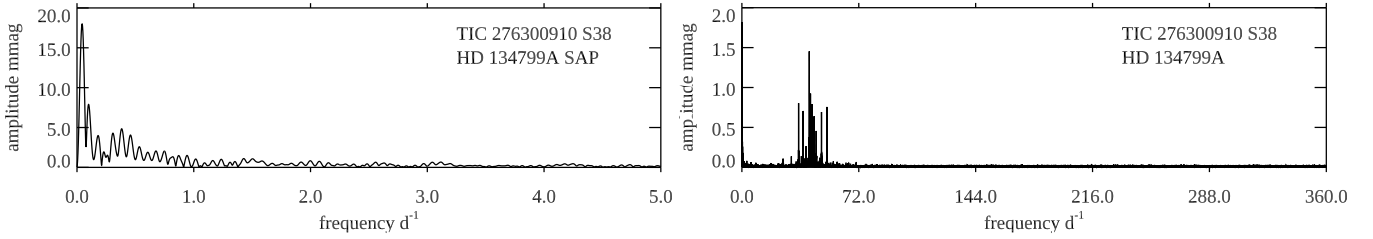
Fig. 5. Amplitude spectra for TIC 276300910 (HD 134799) show clear rotation in the S11 and S38 SAP data with $P_{\rm rot} = 22\overset{\rm d}{.}6$. The low-frequency amplitude spectrum shown here is for the SAP data. This star shows rich multi-periodic δ Sct pulsations, but those probably arise in TIC 276300912 (HD 134799B) which is an A5 star unresolved from TIC 276300910 in the TESS photometry.

For a fifth star, HD 216018 (TIC 36576010), which has resolved magnetically split spectral lines, Mathys (2017) did not detect any significant variation of the mean magnetic field modulus $\langle B \rangle$ nor of the mean longitudinal magnetic field $\langle B_z \rangle$. He argued that, unless the lack of variability results from an infrequent observation geometry (very small angles i or β), the rotation period of the star must be much longer than 6 years. However, combining the magnetic measurements of Mathys (2017) with more recent determinations of $\langle B \rangle$ and a single value of $\langle B_z \rangle$ published by Romanyuk et al. (2016), Giarrusso et al. (2022) proposed a value $P_{\rm rot} = 34\overset{\rm d}{.}044$ for the rotation period of HD 216018. Our own frequency analysis of the magnetic data set of Giarrusso et al. (2022) fails to provide convincing evidence in support of the period value that they advocate. Admittedly, there is a peak close to $f = 0.0293\,{\rm d}^{-1}$ in the amplitude spectrum for $\langle B \rangle$, but there are also other peaks of similar heights at other frequencies, none of which stands out remarkably above the noise. We could not distinguish any significant peak in the amplitude spectrum for $\langle B_z \rangle$. Actually, for this field moment, combining the single measurement of Romanyuk et al. (2016) with those of Mathys (2017) does not add any meaningful constraint as the uncertainty about possible systematic differences between longitudinal magnetic field values obtained with different instrumental configurations may exceed the amplitude of the $\langle B_z \rangle$ curve presented by Giarrusso et al. (2022). Based on these various considerations, we do not regard the $P_{\rm rot} = 34\overset{\rm d}{.}044$ value of the rotation period of HD 216018 as firmly established. Furthermore, two new measurements performed in 2015 yielded values of $\langle B_z \rangle$ close to 1.1 kG (Romanyuk, priv. comm.). The difference of nearly 300 G between these values and the one obtained in 2009 by the same group, with the same instrument (Romanyuk et al. 2016), is highly significant. The 2015 $\langle B_z \rangle$ values are also well outside the range of variation shown in Fig. 39 of Giarrusso et al. (2022) assuming $P_{\rm rot} = 34\overset{\rm d}{.}044$. This strongly suggests that the very long period alternative proposed by Mathys (2017) is the correct one. Accordingly, we kept HD 216018 in the list of candidate ssrAp stars.

For two other stars of Table 2, period values inconsistent with super-slow rotation can be found in the literature. Wraight et al. (2012) reported photometric variations of HD 48953 (TIC 372617495) with a period of $P = 2\overset{\rm d}{.}8939$ from the analysis of photometric observations obtained with the STEREO satellites. However, they note that the observations are affected by blending from another star and systematic effects, and that the variability signal is weak, so that the value of the period should be taken with caution. The TESS data for this star do not show evidence of significant variability, even at a very low level. Thus, we regard HD 48953 as a bona fide ssrAp star candidate.

For HD 66318 (TIC 410451752), the amplitude spectrum computed for the S34-37 data shows a 55σ peak at frequency $f = 1.287\,{\rm d}^{-1}$. This is consistent with the value of the period $P = 0\overset{\rm d}{.}77688$ derived by Cunha et al. (2019) from analysis of S1 data. However, David-Uraz et al. (2019) note that the signal of this star is highly contaminated, so that there is a significant probability that the observed variations correspond to another star. This appears all the more plausible since HD 66318 shows resolved magnetically split spectral lines (Bagnulo et al. 2003), whose very clean component profiles cannot easily be reconciled with a rotation period that, at $P_{\rm rot} = 0\overset{\rm d}{.}78$, would be one of the shortest known for any Ap star. It is much more probable that HD 66318 is a ssrAp star. If the $f = 1.287\,{\rm d}^{-1}$ peak does arise from HD 66318, it is thus likely to be from a g mode.

Besides the five stars already mentioned, a sixth ssrAp candidate from Table 2 shows resolved magnetically split lines: HD 3988 (TIC 262956098). Its rotation period is likely (very) long, as is the case for the majority of Ap stars with magnetically resolved lines (Mathys 2017).

Determinations of $v \sin i$ were found in the literature for five of the stars of Table 2 in which the magnetic resolution of spectral lines has not been observed until now and no estimate of the rotation period (or of a lower limit) has been found in the literature. For four of them, the published values of $v \sin i$ are consistent with the non-detection of any rotational broadening at the spectral resolution of the instrument used for the observation. However, the value $v \sin i = 38\,{\rm km\,s}^{-1}$ reported by Zorec & Royer (2012) indicates that HD 155102 (TIC 405557056) is not a slow rotator. The broadening of its metal lines can be easily seen in Fig. 1 of Catanzaro (2006) by comparing them with the lines of lower $v \sin i$ stars of this study. The lack of variability in the TESS observations of HD 155102 suggests that its rotation and magnetic axes must be nearly aligned. However, according to Catanzaro (2006), while the Si peculiarity type is confirmed by the Si overabundance that they determine, the latter is mild, and one may wonder if the related brightness contrast of the spots is too low to produce detectable photometric variations in integrated light. The same doubt arises about another star listed in Table 2, HD 117227 (TIC 438694338), which has similarly broad lines, as seen in a FEROS (Fiber-fed Extended Range Optical Spectrograph) spectrum ($R \simeq 48\,000$) of our collection. In any event, while HD 117227 and HD 155102 qualify as ssrAp star candidates according to the selection process described in Sect. 2, inspection of a FEROS spectrum of the former and consideration of a meaningful published value of $v \sin i$ for the latter show that they are definitely not ssrAp stars. This stresses the importance of using spectroscopy to confirm the slow rotation of the candidates identified with TESS. However, the other examples discussed above show that spectroscopic confirmations are much more

frequent than the opposite. This validates our search method on a statistical basis.

5.2. The moderately long period stars

As discussed in Sect. 2.4, the initial selection of candidate ssrAp stars based on the analysis of the PDCSAP included a number of stars that actually undergo significant photometric variations over the 27-d duration of a TESS sector. These variations are removed together with the instrumental effects in the data processing but can be seen in the SAP data and characterised from them. Most of these stars have periods in the range $20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$. We identified eight such stars in the Cycle 3 and Cycle 4 samples; they are listed in Table 3.

As already noted, one of them, HD 97394 (TIC 162027140) has resolved magnetically split lines (Elkin et al. 2011). For two more, we found $v \sin i$ determinations in the literature: HD 122525 (TIC 312839768; $v \sin i = 7\,\mathrm{km\,s^{-1}}$; Ryabchikova & Romanovskaya 2017) and HD 185256 (TIC 96855460; $v \sin i = 5.5\,\mathrm{km\,s^{-1}}$; Kochukhov et al. 2013). Furthermore, HD 40678 (TIC 427032468) shows very sharp, unresolved spectral lines in a FEROS spectrum of our collection. We could not find any information about the line profiles for the other four stars of Table 3. However, in the range of rotation periods that they span, some may possibly show resolved magnetically split lines. Unfortunately, magnetic field estimates are only available for three stars of Table 3, HD 97394, HD 122525 and HD 185256, whose line profiles are also well characterised.

6. Conclusion

In Paper I, we had identified 60 Ap stars for which TESS data did not show any variability. However, FEROS spectra from our collection showed that six of them have broad or very broad lines. This left 54 ssrAp star candidates. Sixty-seven new candidates were added in Paper II. Nevertheless, three of them had well-established rotation periods comprised between 26 d and 49 d. All three undergo significant photometric variations over the 27-d duration of a TESS sector. These variations had not been detected in our systematic search. The reason why they were overlooked was unambiguously identified in the present study: part of the physical signal coming from the stars is lost in the data processing carried out to generate the PDCSAP reduced data on which our analysis is based. The usage of the latter for our purpose is justified, so as to deal with instrumental effects. Nevertheless, the non-processed SAP data can still be exploited to extract valuable information about the rotation of stars whose brightness modulation is large enough with respect to the TESS instrumental effects, especially for those with $20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$. Eight such stars were identified in this paper, for which no variability had been detected in the PDCSAP data.

The systematic analysis of the SAP light curves that led to the detection of the variations has not been carried out for the ssrAp star candidates of Papers I and II. The latter will include a number of stars with moderately long periods ($20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$), which are not ssrAp stars as per the original definition of Mathys (2020). In fact, three stars with periods in this range are definitely present in Table A.1 of Paper II. More such periods may be constrained from consideration of the SAP data for the other ssrAp star candidates of Papers I and II. This is an issue outside the scope of the present work, but we plan to address it in a future study. Suffice it to say here that the inclusion or exclusion of the stars with $20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$ in the samples of ssrAp star candidates should be expected to have a minor impact

on the statistical conclusions that can be reached by analysing these samples. As already stressed above, Ap stars with $P_{\mathrm{rot}} \gtrsim 20\,\mathrm{d}$ are indeed part of the long-period tail of the distribution of the Ap star rotation anyway, so that they are relevant for the understanding of the physical mechanisms responsible for, and related to, slow rotation. Thus, at present, conclusions can be best drawn from consideration of the combined list of 33 stars of Tables 2 and 3 with the ssrAp candidate lists of our previous two papers.

Another source of potential contamination of the list of ssrAp star candidates from our TESS Cycle 1 and Cycle 2 studies whose criticality was emphasised in the present work on Cycles 3 and 4 is the misclassification of a number of stars. Namely, we rejected 31% of the stars of Cycle 3 that we had originally selected on the basis of their classification in the Renson catalogue, for which there was no classification uncertainty flagged in this catalogue, on account that they definitely were not Ap stars or they did not appear to be bona fide Ap stars. For Cycle 4, the rejection fraction was even much higher, 72%, but the difference can be in large part attributed to the unreliability of the Ap classifications of a single study (Chargeishvili 1988), which we established. Leaving this source apart, the rejection rate for Cycle 4 would have been 37.5%, similar to that for Cycle 3.

The vetting that we performed involves a certain amount of subjectivity, and we erred on the side of conservatism to minimise the potential contamination of the final list of non-variable Ap stars by misclassified stars of other types. But we do not expect more than a minority of the stars that we did not regard as bona fide Ap stars to have their peculiarity eventually confirmed.

In the construction of the list of ssrAp star candidates of Paper II, we had already tried to include only bona fide Ap stars. To this effect, we excluded the stars that were not in the Renson catalogue from the selection (except for a few for which compelling evidence of peculiarity had been subsequently found), and we carried out a critical evaluation of the peculiarity of the stars flagged with '/' or '?' in the first column of the Renson catalogue. While some additional stars for which the information from the catalogue does not suggest any classification ambiguity were also discarded on account of clear indications of misclassification in more recent publications, this was not part of a systematic vetting such as the one described in this paper for the Cycle 3 and 4 stars. Thus, the ssrAp candidate list for Cycle 2 may still contain a small number of non-Ap stars. By contrast, the search for ssrAp candidates using the TESS Cycle 1 data included all the stars with an Ap classification found in the TIC for which 2-min cadence observations were available. Accordingly, one must expect the list of ssrAp candidates of Paper I to be significantly contaminated by misclassified normal A stars, Am stars, and stars showing other peculiarities, among others.

Notwithstanding the above-described differences in the critical evaluation of the ssrAp star candidate lists of Paper I, Paper II, and this paper, these lists can be combined to draw meaningful statistical conclusions. The merged list contains 152 stars that are either ssrAp star candidates or Ap stars with moderately long periods ($20\,\mathrm{d} \lesssim P_{\mathrm{rot}} \lesssim 50\,\mathrm{d}$). The eight stars for which no TESS variability was detected but that are known to show rotationally broadened spectral lines are not included in this number.

In addition to these 152 stars, there are 28 known ssrAp stars in Table 1 that were not identified as part of our search for Ap stars showing no low-frequency photometric variability
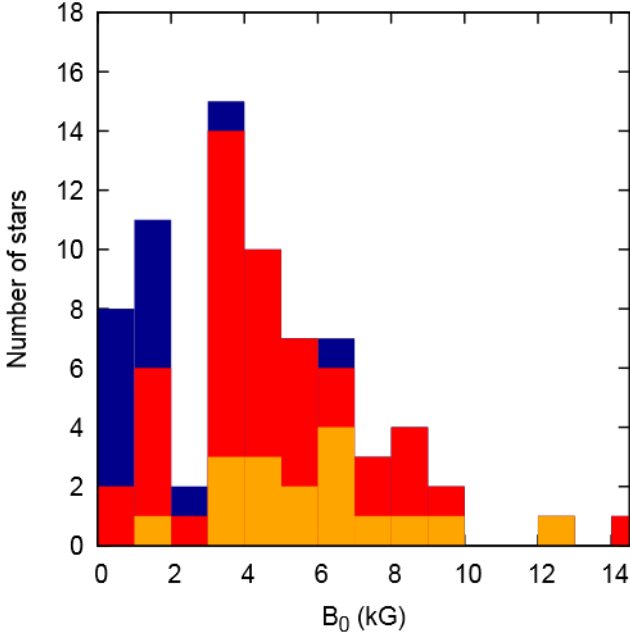
Fig. 6. Distribution of the phase-averaged magnetic field strength B_0 for the long-period Ap stars of Tables 1–3 of this paper, of Table 1 of Paper I and of Table A.1 of Paper II. The orange and red parts of the histogram correspond to the stars for which measurements of the mean magnetic field modulus or of the mean quadratic magnetic field are available; for the remaining stars (blue part of the histogram), a lower limit of B_0 was inferred from the existing mean longitudinal magnetic field measurements. Orange distinguishes the known ssrAp stars that were not identified as ssrAp star candidates on the basis of our TESS-based photometric survey.

over 27 days in the TESS Cycle 1–4 data that we analysed. The reasons why these stars were missed by our search include the non-availability of 2-min observations for some stars, contamination of the signal by another, variable star (either a physical companion or a chance alignment), or instrumental effects. For statistical purposes, these 28 known ssrAp stars can valuably be added to the 152 ssrAp star candidates (or Ap stars with moderately long periods) identified from analysis of the TESS data. This increases the size of the sample to 180 stars.

Of these 180 (very) slowly rotating Ap star candidates, 38 are roAp stars. That is, roAp stars account for 21% of the Ap stars whose rotation periods are in the long-period tail of the distribution. The difference with the fraction of roAp stars among the entire population of Ap stars studied in TESS Cycles 1 and 2, which is of the order of 5.5% (Holdsworth et al. 2021, 2024), is highly significant. The possible inclusion in the long-period candidate lists and in the full Ap star population list of stars whose classification is mistaken and that are not actually Ap stars does not challenge this conclusion (even if every second star of the full Ap star population were misclassified, the roAp star fraction in this sample would only reach 11%). Its implications for our understanding of the origin of rapid oscillations and of (very) slow rotation are currently unclear. Their discussion is beyond the scope of this paper.

For 12 of the 33 stars of Tables 2 and 3, at least some magnetic field determinations are available in the literature. They can be used together with the 42 ssrAp star candidates from Papers I and II for which magnetic field measurements exist to confirm and refine the conclusions previously reached about the distribution of the magnetic field strengths in (super) slowly

rotating Ap stars. The size of the sample can be further increased by adding to it those stars that are definitely known to be ssrAp stars but that were not identifed as ssrAp star candidates from consideration of the TESS Cycle 1 to Cycle 4 data. There are 15 such stars that show resolved magnetically split lines, for which mean magnetic field modulus measurements have been obtained. Furthermore, one determination of the mean quadratic magnetic field of HD 101065 has been performed. Thus, in total there are 70 known ssrAp stars, ssrAp star candidates, or Ap stars with a moderately long period for which magnetic information is available. Since non-Ap stars do not have large-scale organised magnetic fields, the probable presence of a number of them in Table 1 of Paper I and in Table A.1 of Paper II is irrelevant for the present discussion.

Figure 6 shows the distribution of the magnetic field strength B_0 for the stars of Tables 2 and 3 of the present paper, Table 1 of Paper I and Table A.1 of Paper II, as well as for the stars of Table 1 that have not been identified as ssrAp star candidates as part of our TESS-based survey. For those stars with resolved magnetically split lines for which mean magnetic field modulus values have been derived, B_0 is the average of these values, over a rotation cycle if one at least has been covered, or over the available measurements otherwise. As in Paper II, when no mean magnetic field modulus determinations are available but mean quadratic magnetic field data have been obtained, B_0 was approximated by dividing the phase-averaged value Q_0 of the mean quadratic field by 1.28. Finally, for those stars for which only the mean longitudinal magnetic field has been measured, the lower limit of B_0 was set to 3 times the rms mean longitudinal field $\langle B_z \rangle_{\rm rms}$; this is the value that was used in Fig. 6.

For interpretation of this figure, the caveats discussed in Paper II remain relevant. In particular, among the 12 ssrAp star candidates or moderately long-period Ap stars of the present paper for which magnetic field measurements exist, there is none for which those measurements sample a whole rotation cycle. Accordingly, the phase-averaged field values listed in Tables 2 and 3, which were used to build Fig. 6, sample only those rotation phases at which observations have been obtained. While the resulting values of B_0 and Q_0 should arguably be reasonable first approximations of the average values over a full rotation cycle of the mean magnetic field modulus and the mean quadratic magnetic field, respectively, the value of $\langle B_z \rangle_{\rm rms}$ based on incomplete phase coverage may be considerably less representative of the actual strength of the mean longitudinal magnetic field (see Paper II for details). This is why, in Fig. 6, the parts of the histogram based on $\langle B_z \rangle$ measurements are filled in blue, to distinguish them from those based on $\langle B \rangle$ or $\langle B_q \rangle$ data. These are filled in red for the ssrAp star candidates, and in orange for those known ssrAp stars that were not identified as photometrically constant over one TESS sector. The distribution of the latter supports the view that the knowledge of the ssrAp stars prior to our current systematic TESS-based search was biased towards strongly magnetic Ap stars, but it is otherwise fully consistent with the distribution derived from consideration of the ssrAp star candidates from the current project.

Figure 6 is an updated version of Fig. 4 of Paper II, which mostly confirms the conclusions drawn from consideration of the latter. First and foremost, it confirms that the rate of occurrence of (super) slow rotation among weakly magnetic Ap stars must be considerably lower than among strongly magnetic ones. Not only are there fewer stars in the 0−3 kG B_0 range than in the 3−6 kG B_0 range, but furthermore this trend is opposite to that found by other studies that consider samples of Ap stars with any rotation periods, as discussed in detail in Paper II.

The intriguing gap in the B_0 distribution between 2 and 3 kG, already suggested by Mathys (2017), also appears to be confirmed. Admittedly, compared to Fig. 4 of Paper II, one star, HD 3988, was added in this histogram bin, on the basis of its mean magnetic field modulus, $B_0 = 2.65$ kG. However, the four spectra from which this value was derived were acquired over a time interval of only five days, and they show no significant $\langle B \rangle$ variations. This is not surprising for an ssrAp star candidate, but it implies that the actual average of the mean magnetic field modulus of HD 3988 may differ from the value of B_0 reported here. Even if it remains between 2 and 3 kG, the presence of HD 3988 in this histogram bin would not rule out the existence of a gap in the magnetic field strength distribution. As already mentioned in Paper II, the presence of this gap suggests the existence of two distinct populations of ssrAp stars, consisting of weakly and strongly magnetic stars whose rotational evolution would follow different paths and involve different physical mechanisms. This potentially has major implications for the understanding of the origin of the magnetic Ap stars, which fully justify further investigation of super-slow rotation in these stars. In particular, it will be essential to constrain the rotation periods and the magnetic fields of all ssrAp star candidates so as to establish their distribution on the basis of a sample as complete and unbiased as possible.

Perhaps the most remarkable difference between Fig. 6 and Fig. 4 of Paper II is the presence in the former of the star HD 66318, with $B_0 = 14.5$ kG, which is well outside the range covered by the rest of the sample, in which no ssrAp star candidate has $B_0 > 10$ kG. We expect very few ssrAp stars with values of the mean magnetic field modulus of the order of that of HD 66318, since until now no star with $P_{\rm rot} > 150$ d has been known to have $B_0 > 7.5$ kG (Mathys 2017). In particular, the rotation period of the other star in Fig. 6 for which $B_0 > 10$ G, the well-known ssrAp star HD 126515 ($B_0 = 12.5$ kG), is $P_{\rm rot} = 129\overset{\rm d}{.}95$. Bagnulo et al. (2003) obtained three measurements of the mean magnetic field modulus of HD 63318, the second one 49 days after the first one and the third one 11 days after the second one. The fact that all three measurements yield essentially the same value of $\langle B \rangle$ does not necessarily rule out a rotation period shorter than 150 d. Nonetheless, determining the magnetic field of HD 66318 at more epochs in order to constrain its rotation period would represent a valuable test of the non-occurrence of very strong fields in stars with periods in excess of ~150 d.

In summary, the addition of the new ssrAp star candidates identified from analysis of the TESS Cycle 3 and 4 data to those from Cycle 1 and 2, complemented by the inclusion of the known ssrAp stars for which the analysed TESS photometric data do not reveal the lack of variability over 27 d, confirms and strengthens the conclusions reached in Papers I and II about the dependence of the rate of occurrence of rapid oscillations and of magnetic fields of different strengths on the rotational velocity of Ap stars. To consolidate these conclusions, the following steps need to be carried out. (1) The list of ssrAp star candidates from Papers I and II must be reviewed for overlooked variability possibly detectable in the SAP data but removed in the processed PDCSAP data and for possible misclassifications of non-Ap stars as peculiar. (2) High-resolution spectra of all ssrAp star candidates for which such data are not available yet must be obtained to confirm that they have low projected equatorial velocities and to determine their magnetic fields. We are now undertaking these steps, whose results will be reported in future papers.

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Appendix A: Incorrect and dubious Ap classifications

As mentioned in Sect. 2.2, a critical review of the information available in the literature led us to conclude that the Renson catalogue includes a significant number of stars assigned an Ap spectral type that either are definitely not Ap stars or that cannot be regarded as bona fide Ap stars, without any indication of the ambiguity of their classification. For some of these, but not all, the classification was revised or questioned after publication of the catalogue. Following this realisation, we undertook a systematic literature review to confirm as securely as possible that all the stars that we identified as ssrAp star candidates on account of the lack of photometric variability in the TESS Cycle 3 and Cycle 4 data are indeed bona fide Ap stars. As a result, we discarded a number of stars from our initial selection.

Although our review covered only a small fraction of the Ap stars featuring in the Renson catalogue, we believe that the list of those among them that are definitely not Ap stars or that we do not consider as bona fide Ap stars represents a useful piece of information for the community. There are two groups of such stars.

The first one consists of those stars for which the only reference that we could identify as the source of the Ap classification is the Abastumani catalogue (Kharadze & Chargeishvili 1990). We showed in Sect. 2.2 that more than 50% of the Ap stars listed in this catalogue must be misclassified. Therefore, stars in this group cannot be regarded as bona fide Ap stars, unless some other piece of evidence can be found that confirms their classification – for instance, the detection of variations that may either be due to rotation or to rapid oscillations. No such piece of evidence was found for 49 of the 51 stars of this group that were present in our initial automated selection of stars that show no photometric variability over the duration of a TESS sector.

These 49 stars are listed in Table A.1. A number of entries in this table must definitely correspond to Ap stars, but on an individual basis, their peculiarity needs to be independently confirmed.

The second group of stars with incorrect or dubious classifications as peculiar contains a few stars for which we were unable to identify the source of the Ap spectral type as well as stars for which we found references convincingly indicating a different spectral type or (in two cases) an ambiguous classification. The stars of this group are listed in Table A.2, which contains 14 entries. These stars were all identified by the automatic search procedure as showing no TESS photometric variability over the 27 d duration of a sector; for one of them, TIC 152803574, the analysed TESS observations were obtained both in Cycle 3 and in Cycle 4.

Table A.1. Stars listed as Ap in the Renson catalogue, for which the Abastumani catalogue is the only identified source of classification as peculiar. Columns 1 and 2 contain the TIC number and another identifier; the spectral type from the Renson catalogue is given in Col. 3.

TIC	Other ID	Spectral type
3373254	HD 244248	A5p SiSr
3542929	HD 244372	A1p Sr
3544794	HD 244352	B9p Si
20077256	HD 246148	B9p Si
73904395	J05312310+2814129	B9p SiSr
74387907	TYC 1858-556-1	B9p SiSr
74804951	HD 245423	A3p Si
74805804	HD 245353	A1p SiSr
75777460	HD 246587	A2p SiCr
75856090	HD 246686	A1p Si
76031807	HD 246861	B9p Si
76098926	HD 246993	B9p Si
76213603	TYC 1870-1678-1	A0p Sr
77834479	HD 248663	B9p Si
78239007	HD 248944	A1p SiSr
78789635	HD 40038	A0p Sr
78968090	HD 249664	A1p SiSr
79042758	TYC 1867-2081-1	B9p SiSr
79322287	J06002427+2842163	A0p Si
79963265	TYC 1872-1791-1	B9p SiSr
80895911	HD 41418	A0p Si
80896249	HD 251408	A1p SiCr
81436350	HD 252286	A0p Sr
115453793	HD 244955	Ap Sr
115633147	HD 245191	A0p Si
116143102	HD 245725	A0p Si
116247916	TYC 2408-446-1	B9p SiSr
127712498	TYC 2411-2277-1	B9p Si
127837999	TYC 2407-384-1	B9p SiSr
127838106	TYC 2407-631-1	A0p SiSr
127960970	HD 243523	A0p SiSr
239758529	HD 247437	B9p SiSr
239802019	HD 247629	A0p Sr
239807112	TYC 2405-821-1	B9p SiSr
239833172	HD 247794	A0p Sr
285856246	HD 243791	A0p Sr
308704616	TYC 2409-31-1	B9p SiSr
309118102	HD 249382	A0p Si
400163968	HD 250149	B9p SiSr
429127816	TYC 1855-373-1	A0p SiSr

Table A.2. Stars listed as Ap in the Renson catalogue, with an incorrect or dubious classification as peculiar. Columns 1 to 3 give the TIC number of the star, an alternative identification, and the spectral type listed in the Renson catalogue. The reason why we do not consider the star as a bona fide Ap star appears in Col. 4 (in a number of cases, under the form of a different spectral type), with the corresponding reference (when applicable) in Col. 5.

| TIC	Other ID	Spectral type	Exclusion reason	Reference
2854318	HD 242991	B9p Si	B9IV HgMn	Paunzen et al. (2021)
7731470	CD−31 12076	A5p Sr	Unknown classification source	
43068154	HD 228112	A9p Sr	Normal F0 star	Floquet (1970)
88091070	HD 192969	A0p SiMg	A0p Si or perhaps Am	Bond (1970)
127506807	HD 281048	B9p SiSr	Unknown classification source	
152803574	HD 104833	F0p Sr	F str λ 4077	North et al. (1994)
156645202	HD 111702	F0p Sr	F str λ 4077; composite?	North et al. (1994)
207018530	HD 45827	A7p SrCrEu	No lines[a]	Babcock (1958)
231461096	HD 56731	B9p Si	kA6hA7mF0(IIIb)[b]	Gray & Garrison (1989)
236328832	HD 144999	A7p Sr	Am	Bidelman (1988)
358467049	CPD−60 944B	B9p Si	HgMn	González et al. (2014)
385552299	HD 23387	A0p CrSi	Ap character questionable[c]	Hui-Bon-Hoa & Alecian (1998)
405220654	BD−14 2015	B9p Sr	B9:Vn	Dworetsky (1975)
410033396	HD 202400	F0p Sr	Ba star	North et al. (1994)

[a]See text (Sect. 2.2). [b]The star is also studied as an Am star in many other references. [c]Conclusion derived from an abundance analysis based on a spectrum recorded at $R \simeq 34,000$.

Appendix B: The amplitude spectra

Figs. B.1 to B.4 shows amplitude spectra for the long-period Ap stars. Each row presents a low-frequency amplitude spectrum showing no rotational variation in the left panel, and a full amplitude spectrum to the Nyquist frequency of $360\,\mathrm{d}^{-1}$ in the right panel, which allows detection of δ Sct or roAp pulsation.
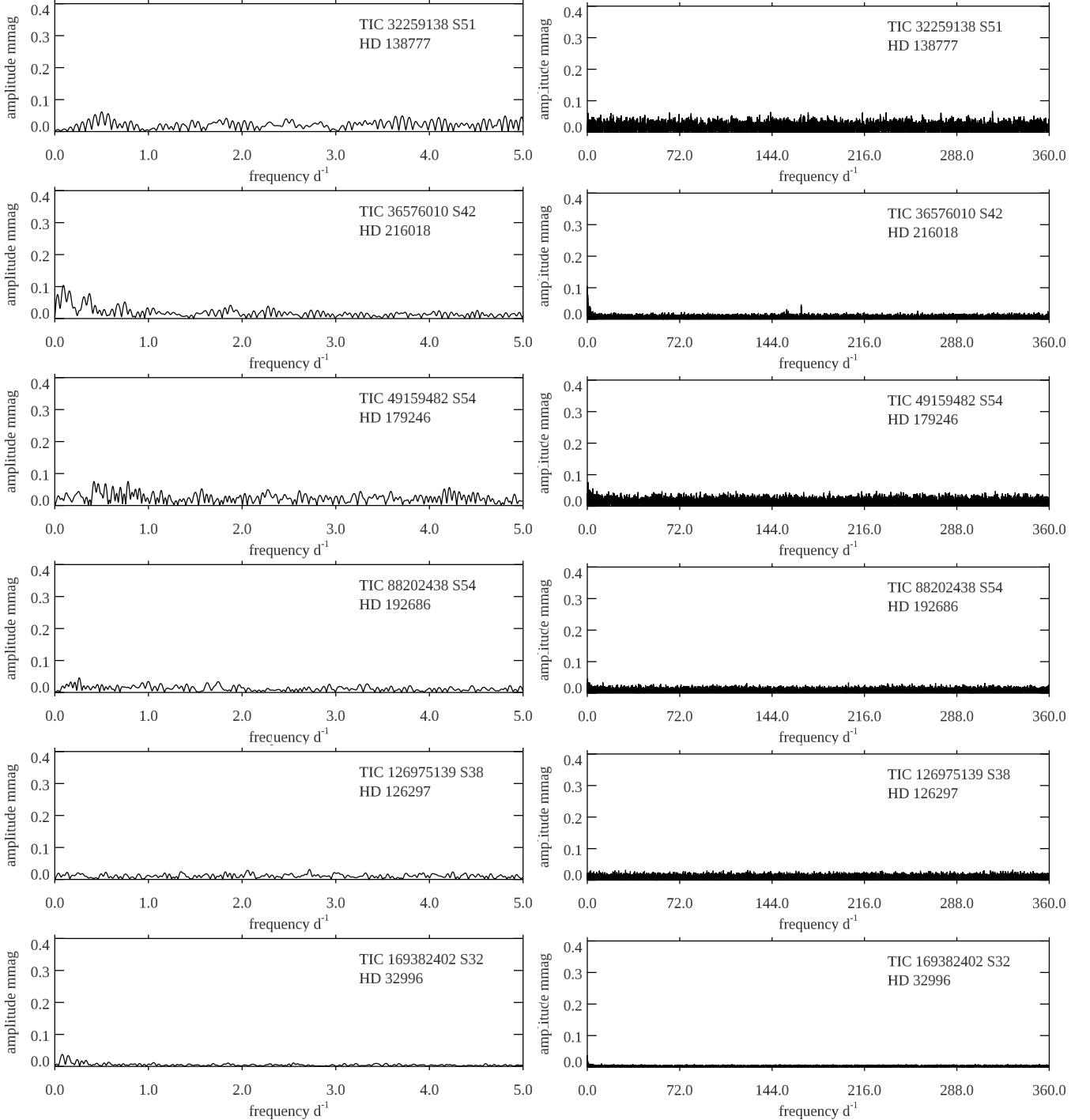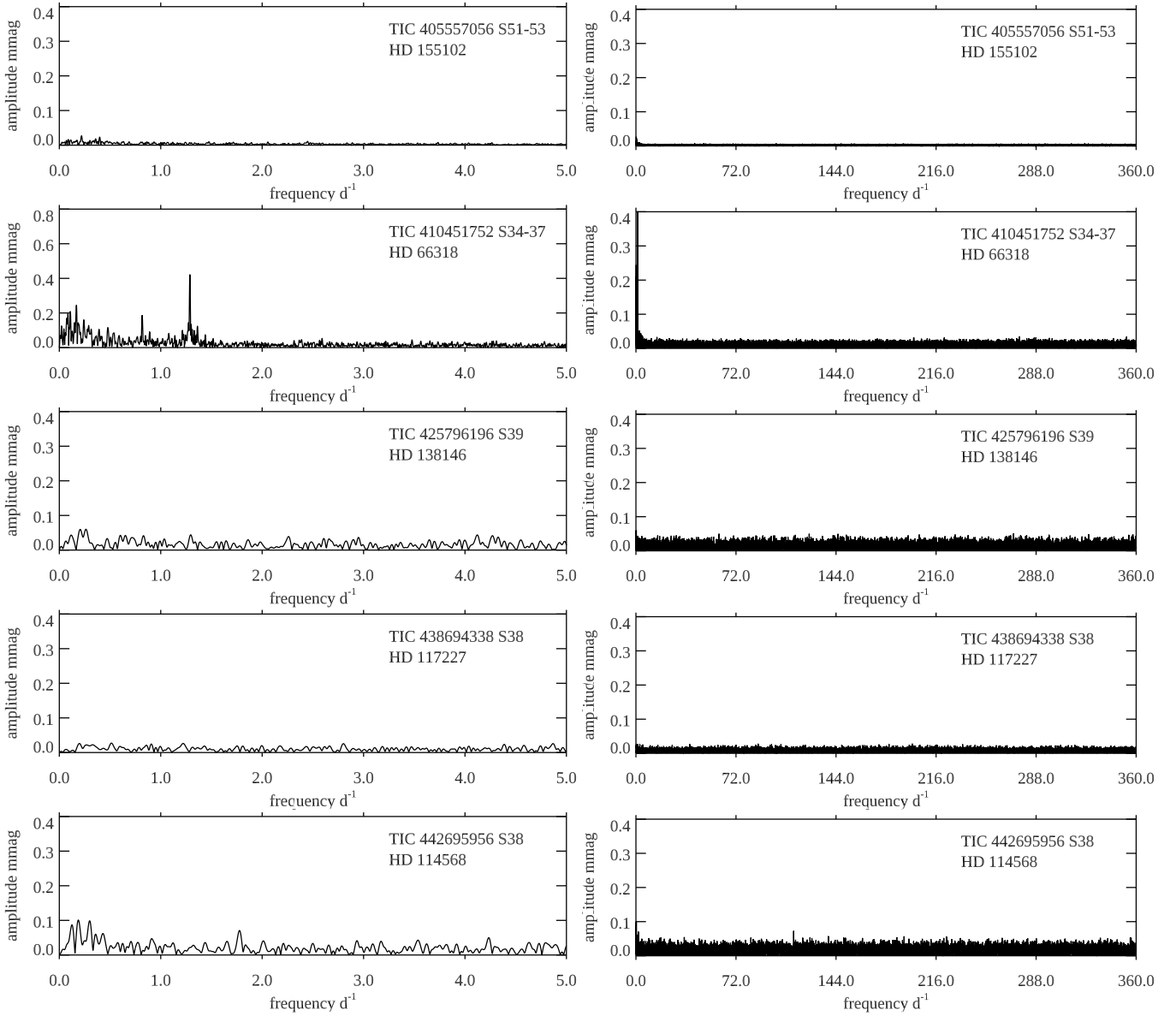

Fig. B.1. Amplitude spectra for the long-period Ap stars. Each row presents a low-frequency amplitude spectrum showing no rotational variation in the left panel, and a full amplitude spectrum to the Nyquist frequency of $360\,\mathrm{d}^{-1}$ in the right panel, which allows detection of δ Sct or roAp pulsation. We note the occasional changes of ordinate scale to accommodate pulsation peaks.
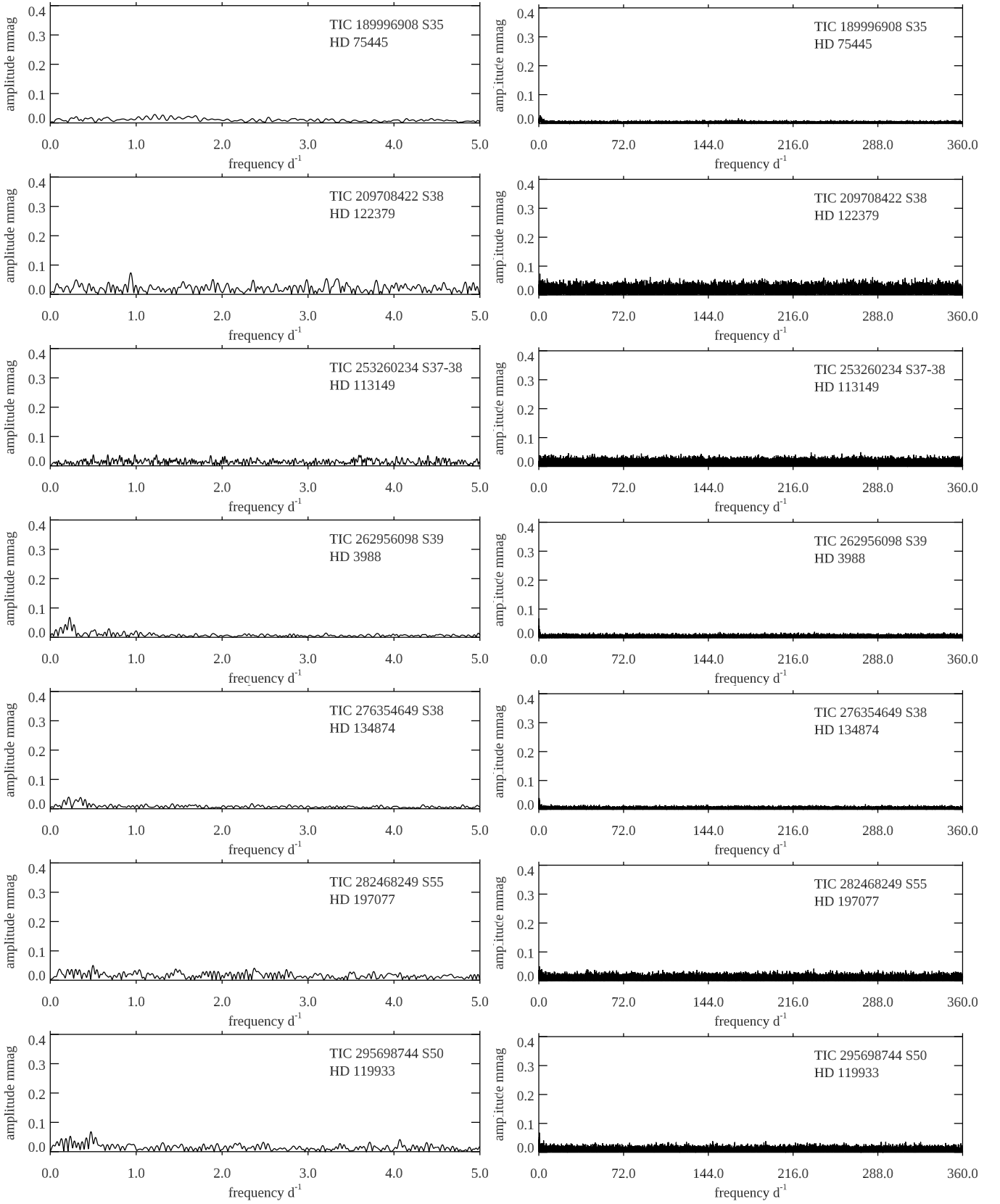
Fig. B.2. Amplitude spectra for the long-period Ap stars – continued.
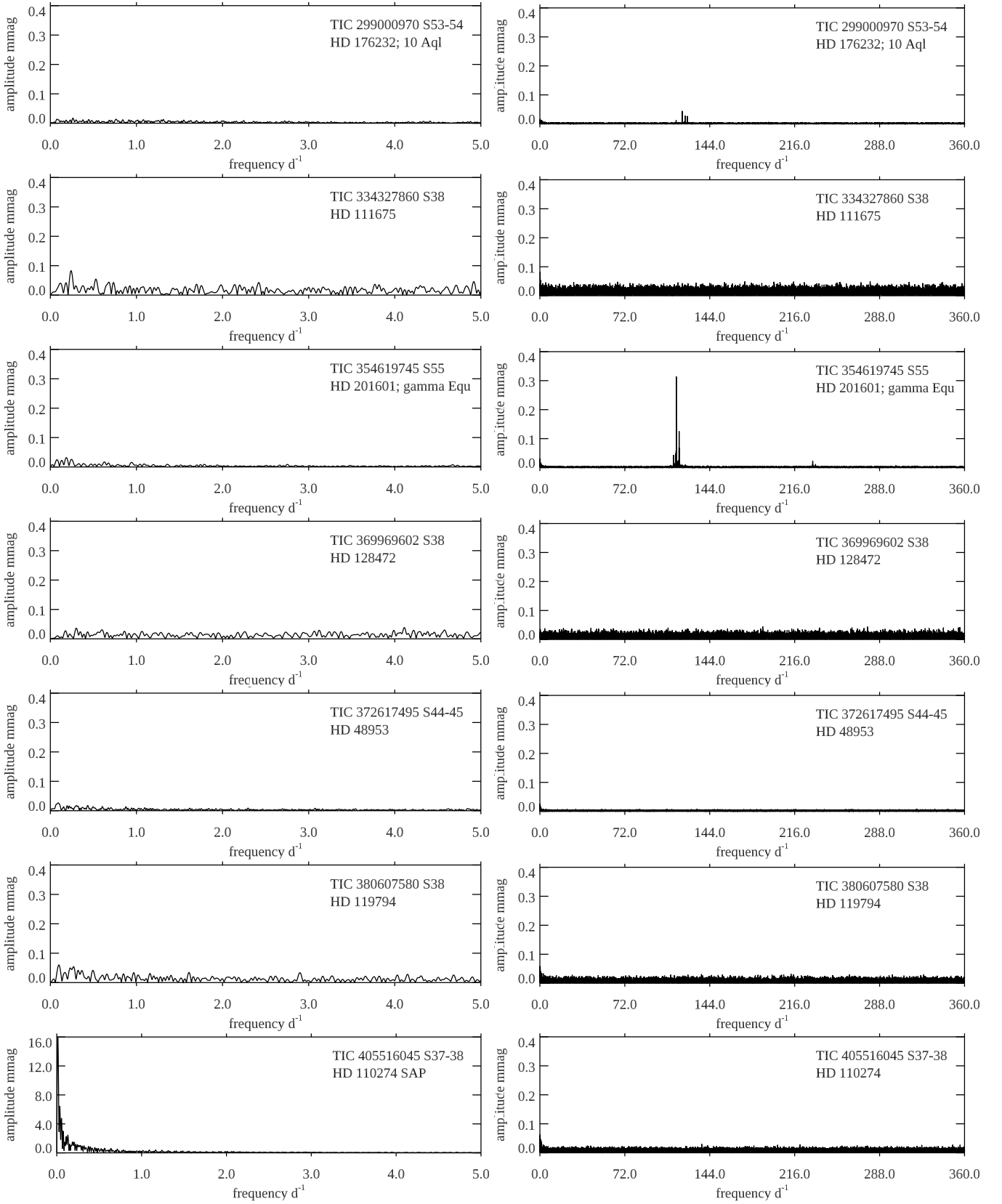
Fig. B.3. Amplitude spectra for the long-period Ap stars – continued.

Fig. B.4. Amplitude spectra for the long-period Ap stars – continued. TIC 410451752 (HD 66318) has a 55 σ peak at 1.2874 ± 0.0001 d^{-1} (P = 0.77675 ± 0.00006 d), and a 24 σ peak at 0.0174 ± 0.0002 d^{-1}. These may be g modes, which is unusual for an Ap star.