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Multi-body Dynamics in Vehicle Engineering

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Abstract

Since Euler's original gyro-dynamic analysis nearly two and a half centuries ago, the use of multi-body dynamics (MBD) has spread widely in application scope from large displacement rigid body dynamics to infinitesimal amplitude elastodynamics. In some cases, MBD has become a multi-physics multi-scale analysis, comprising contact mechanics, tribodynamics, terramechanics, thermodynamics, biomechanics, etc. It is an essential part of all analyses in many engineering disciplines, including vehicle engineering. This paper provides an overview of historical developments with emphasis on vehicle development and investigation of observed phenomena, including noise, vibration and harshness (NVH). The approach undertaken is comprehensive and provides a uniquely focused perspective, one which has not hitherto been reported in the literature.

Keywords

Multi-body dynamics, elastodynamics, tribodynamics, multi-physics multi-scale analysis, vehicle dynamics, powertrain NVH

Early Historical Developments

The history of multi-body dynamics goes back to at least the Bronze Age when the Sumerians in Mesopotamia devised wheels by sticking solid axles through wooden discs. The use of these wheels to move carts/chariots provided the first example of a multi-body vehicle system. The history of the wheel, having been famously reinvented more than 50,000 times, parallels the developments of other mechanisms such as sextants, clocks, cam-followers, bearings, pistons, gears, etc. Most inventions and redesigns come about as the result of a need or an idea, not because of deliberate premeditation and scientific/analytical rigour.

Analysis and design evaluation is a relatively new endeavour since the 16th century and the trials of the polymaths such as Galileo Galilei (1564–1642). In those days, analyses were based on perspectives, geometry as an extension of movements of planets, and a form of synthesis in which Galileo excelled with his designs of machines and mechanisms. He followed in the footsteps of his 15th Century predecessor Leonardo Da Vinci (1452-1519). Leonardo proposed inventions such as his self-drive car ("vehicle") and ornithopter [1]; these being the first examples of multi-body systems in the Middle Ages. However, unlike all the previous polymaths, Galileo understood the concept of force and moment, as in centrifugal force and couple and other basic kinetics. Therefore, Galileo laid down the scientific foundation for the mechanical perspective [2, 3].

In Galileo's time, much of the dynamic analysis was based on observation, now termed as kinematics, which is the measurement of observed phenomena without regard to the underlying cause. It was observed by Kepler in 1609 [4] that all the planets in the solar system revolve around the Sun in elliptical orbits of low eccentricity on the ecliptic plane (the Heliocentric system). This enunciation by Kepler and then Galileo put paid to the favoured ecclesiastical Earth-centred universe proposed by Aristotle (384-322 BC). In any case, the first observed multi-body system (the Solar system) engaged the attention of many in the ancient times through to the Middle Ages [5,6].

The motions of planets around the Sun, as kinematic observations, were published in 1609 as Kepler's three laws of motion [4]. In 1687, Isaac Newton [7] showed that the elliptical planetary motions described by Kepler's laws conform to the underlying dynamics established by Newton's own three laws of motion and that of his law of universal gravitation. With the establishment of rigid body dynamics and its application to planetary motion, Newtonian mechanics was born, together with a proper description of the concepts of force and gravitation. Hence, planetary motion in the solar system became the first thoroughly investigated multi-body system.

Fundamentals of Mechanical Multi-body Systems

In the realms of mechanisms and mechanical systems, the first application of rigid body dynamics was presented by Euler for single gyro-dynamics [8]. With this contribution, it became clear that dynamics of multi-body systems are subjected to constraints imposed by their assembly. This led to the generic formulation of constrained multi-body system dynamics by Lagrange [9]. A combined solution of differential equations of motion and algebraic constraint functions was developed. This approach has become the core of multi-body dynamics analysis. Gammel [10] and Magnus [11] provided improved solutions to Euler's gimbal gyro-dynamics. Hence, the analysis of practical mechanical multi-body systems was born.

Other early developments, enabling the use of practical multi-body analysis, included improvements in matrix manipulation methods [12] for large and sparse matrices [13] and numerical integration methods for "stiff systems" (those with widely-split eigenvalues) [14].

Large displacement-low frequency vehicle dynamics (ride and handling)

In the case of vehicle engineering, multi-body dynamics analysis was initially applied to vehicle ride and handling studies. It has been progressively used with larger degrees-of-freedom (DOF) systems for vehicle suspension analysis in industry using such computer codes as Automatic Dynamic Analysis of Mechanical Systems (ADAMS) and Dynamic Analysis and Design System (DADS). These codes provide automatic generation of differential-algebraic equations (DAE) and their simultaneous solution. Therefore, they are suitable for use in industry primarily for constrained rigid body dynamics, typically of suspension kinematics/dynamics, vehicle ride comfort analysis and vehicle manoeuvres/handling/road holding. Within the spectrum of dynamics' phenomena, such applications may be classified as large displacement low frequency events.

MBD studies were initially used for ride comfort and/or vehicle handling by Segel [15] and McHenry [16]. Other early applications of multi-body theory for vehicle dynamics included [17-20]. With the advent of detailed multi-body codes, large DOF vehicle models were developed with practical complex nonlinear suspension systems, and with the inclusion of tyre models [21-27] for vehicle handling analysis. Complex manoeuvres were presented including lane-changes, slalom motions, J-turns, and cornering and braking [27-29]. Hegazy et al [27] presented a 94 DOF full vehicle model and subjected it to various manoeuvres for assessment of vehicle handling in line with international standards such as International Standard Organisation (ISO) and British Standard (BS). Figure 1 shows

the modelled vehicle undertaking a lateral transient manoeuvre specified by BS AU 230 [30]. Other researchers have considered various vehicle control features such as 4-Wheel Steer (4WS) and Direct Yaw Control (DYC) [31, 32].
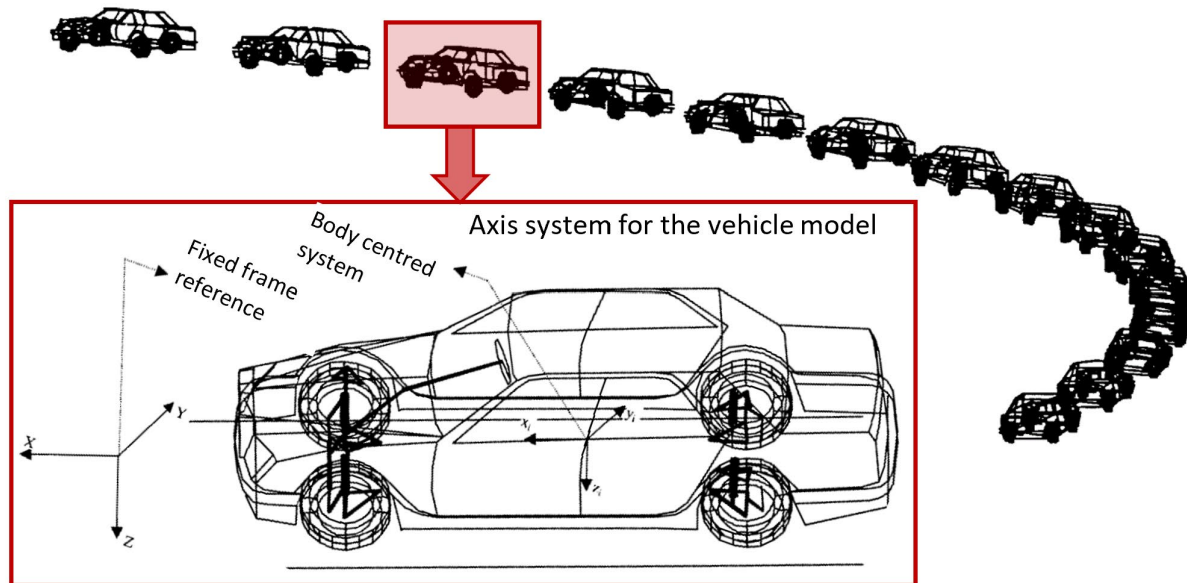

Figure 1: Animated lateral manoeuvre of a full-vehicle multi-body model at 0.8g [30]

Nowadays, multi-body analysis plays an important role in the development and design evaluation of vehicles and their subsystems. Such analyses include suspension kinematics and dynamics, elasto-kinetics, as well as vehicle ride comfort and road holding [19, 33, 34]. Increasingly, a large number of additional load-case scenarios (other than those stated above) are taken into account in vehicle development. These include bounce, pitch and roll in traversing over road calming features (bumps), negotiating single bump events or falling into potholes [35-39]. Harsh conditions such as kerb strike [40] are also considered. von Chappuis et al [40] showed that a simple 7 DOF vehicle dynamics model would suffice for drive-over-kerb studies at various kerb strike speeds of up to 40 kph and for different kerb heights. The model included experimentally verified jounce bumper and rebound stop characteristics (figure 2). The non-linearity and hysteresis of polymeric structures of jounce and rebound stops were included using experimentally verified mapping characteristics rather than the usual approach of simplified representations, using various viscoelastic analytical models. Therefore, fairly accurate prediction of vertical impact reactions of tyres, suspension mounts, jounce bumper and rebound stops were obtained (figure 3). Such studies can help the design of chassis and suspension elements' characteristics.

All these load-case scenarios affect kinematics of suspension, the applied forces acting on the chassis and suspension elements, as well as their structural integrity. Therefore, they can be used for design and design analysis of suspension systems' structures, their integrity and control [41-46]. All these studies necessitate the inclusion of detailed tyre models and elasto-kinetics of suspension compliance.
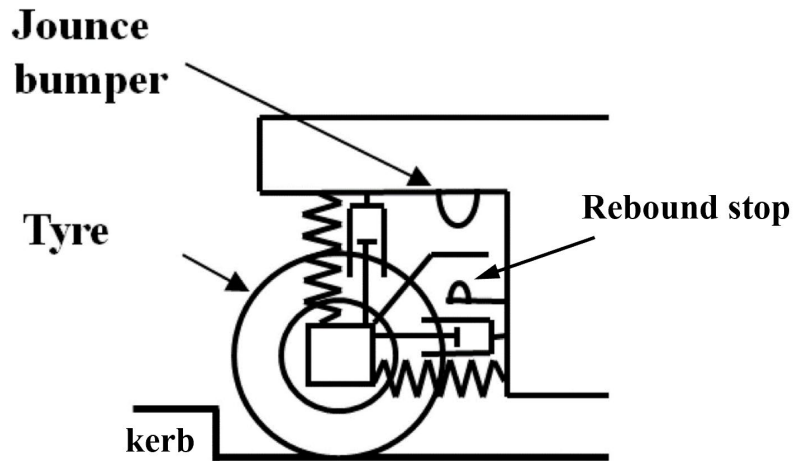
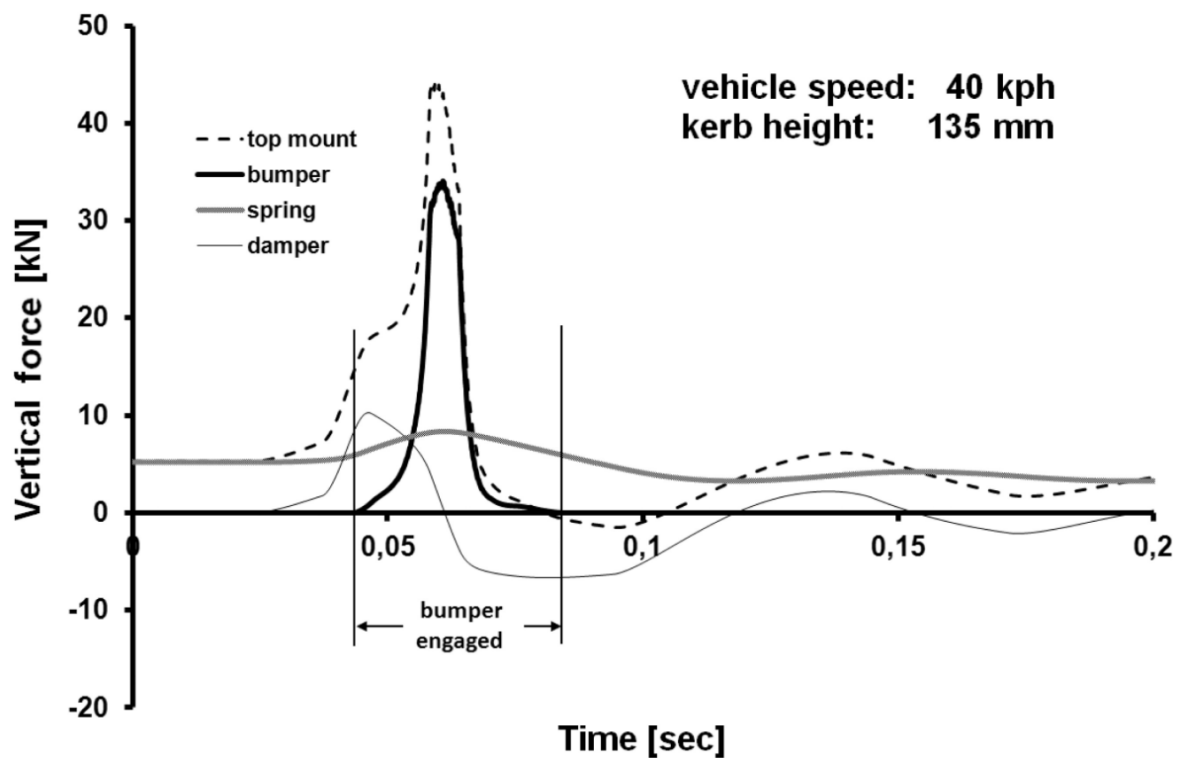Figure 2: A simple vehicle model for drive-over-kerb study [40]

Figure 3: An example of predicted reactions of chassis and suspension elements in drive-over-kerb manoeuvre [40]

Two main types of tyre models have been developed. One is empirical, based on ever-increasing physical testing with purpose-built rigs, detailed testing procedures, data acquisition and regression analysis. Such empirical models lead to the creation of analytical expressions for tyre forces in terms of an increasing number of governing parameters/conditions. Good reviews of tyre modelling

and testing procedures have been provided [47-52]. Among the physical tyre models, the Magic-Formula tyre model is the most widely used [50].

The other type of tyre model is based on fundamental principles of contact mechanics with discretised elemental representation, relating the tyre reaction to localised deformation of the chosen element type. The elements usually include a small mass with stiffness and damping characteristics [47, 53-58]. These elements are generally termed brush elements and may have various elastic or viscoelastic characteristics. Some of these tyre models have been extended to include the effect of tyre carcass elasticity and inflation pressure [57-60]. These additional features make the models more suitable for transient vehicle dynamics with combined slip, cornering and braking under extreme conditions, as well as riding over obstacles and kerb strikes. Figures 4 (a) and (b) show two types of brush models. In both cases, the tyre belt is discretised into small masses in contact with the road surface. Each element is an anisotropic bristle with carcass stiffness and damping properties which may follow any assumed viscoelastic model or simply use measured maps. To improve response characteristics under transient conditions, tyre belt behaviour is represented by elemental interconnections as shown in figure 4(b).
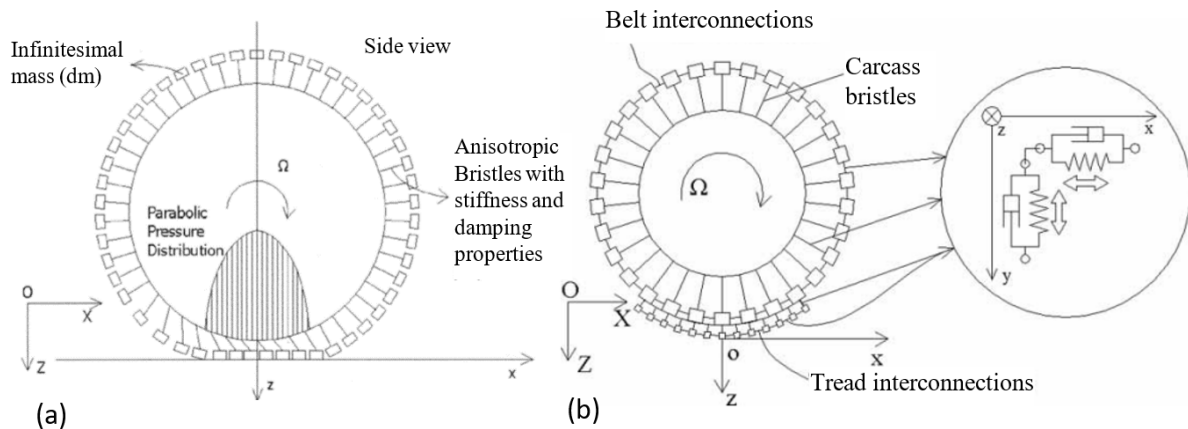

Figure 4: Various brush-type fundamental tyre modes [55, 57]

Figure 5 shows that brush models with belt interconnections better capture the response of the tyre, shown by the fluctuations evident at the onset of any transient manoeuvre. Soft materials such as tyres undergo creep even in rolling with slip, which sets up a transient response. Creep under slip stretches the tyre belt so that its periphery appears to be longer. This effect is captured by belt interconnections in the model of figure 4(b). Belt, carcass and bead flexibility have also been included in some widely used empirical physical tyre models [61, 62].
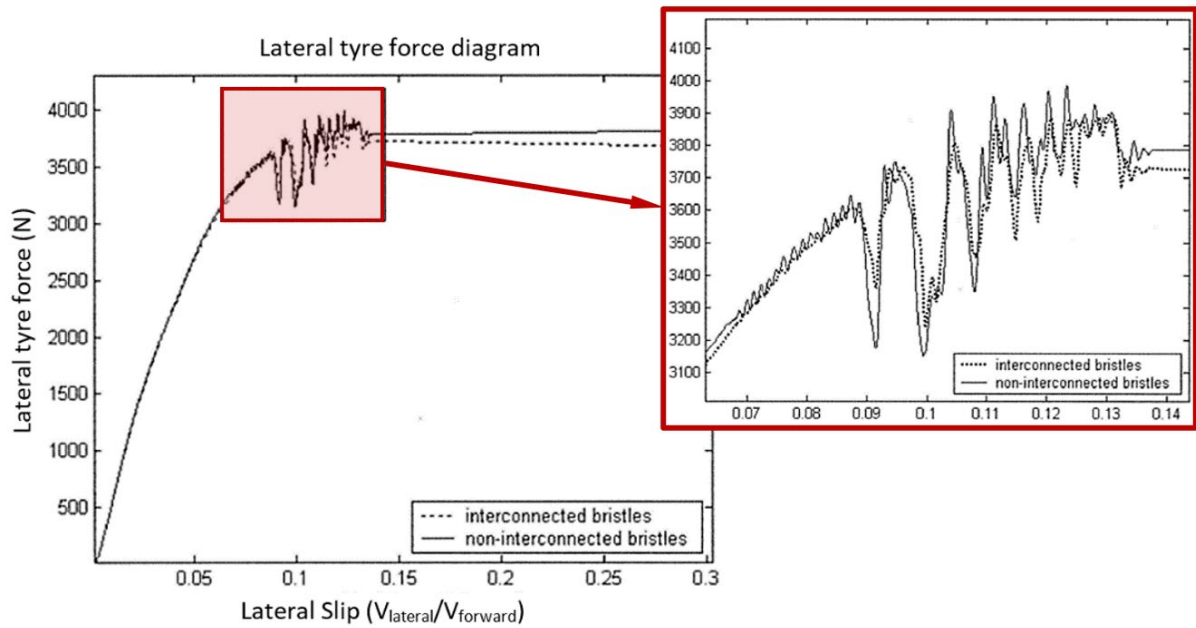
Figure 5: Tyre response characteristics under transient lateral slip conditions [55]

Friction can be implemented between the small-discretised masses and the road surface in order to represent tyre-paved road contact in the case of brush-type models. Coefficient of friction (rolling resistance) can be estimated based on an iterative recursive solution, or stick-slip motion, or different road surface topography, as well as on any changes in environmental conditions [54-56, 63-68]. Therefore, conditions leading to slipping can be investigated for vehicles travelling over terrains with some wheels running on unmade road verges, or ice or snow. Such conditions cause drag or slip., and are referred to as split-μ [69]. They can cause directional stability issues and tyre transience, when steering and braking, that require various control systems such as anti-blocking braking system (ABS), and four-wheel steer (4WS) [70-74].

Off-road vehicles often run on soft and deformable surfaces (unprepared terrain). A detailed analytical method was proposed by Bekker [75, 76] for the parametric analysis of off-road vehicle performance. The most important aspect of off-road vehicle dynamics is the inclusion of pertinent tyre models. Soil sinkage under a wheel is caused by two independent effects: the pressure-sinkage orthogonal to the soil surface, and shear-tension-displacement along the direction of a rolling wheel (ruttage). Terramechanical tyre-soil interaction include either phenomenological analytical models with viscoelastic or poroelastic representation of the soil, or numerical models using finite element analysis [77-83]. Various studies with off-road multi-body vehicle dynamics have been carried out on different soft soils such as sand and loam. This is in order to improve the tractive efficiency and optimisation of drivetrain components such as differentials and visco-lock systems [80, 84-88].

Within the area of vehicle handling, some other features can include the effect of aerodynamic forces (lift and drag), as well as unsteady conditions such as crosswind and side gusts [89-95]. Hussain et al [91] used the same detailed 94 DOF full-vehicle model in [27] to study the same cornering manoeuvre, but with the introduction of aerodynamic forces. The lift and drag characteristics of the vehicle were obtained using a 10% physical scale model placed in a wind tunnel prior to analysis using Computational Fluid Dynamics (CFD). The aerodynamic and multi-body vehicle handling co-simulation was compared with the same vehicle manoeuvre without the inclusion of

aerodynamic action. Most modern saloon vehicles corner up to a lateral acceleration of 0.8g, during which a body roll angle of 2°-8° occurs. Figure 6(a) shows the body roll during the cornering manoeuvre with and without the inclusion of aerodynamic forces. In both cases the body roll increases prior to reaching the corner and levels-off after the corner has been negotiated. During such a manoeuvre, body weight transfer occurs from the inside to the outside wheels. It can be seen that the aerodynamic lift reduces the lateral weight transfer. One practical implication of such studies is that aerodynamic forces improve the vehicle stability in such motions with reduced lateral acceleration for the same manoeuvre as is shown in figure 6(b).
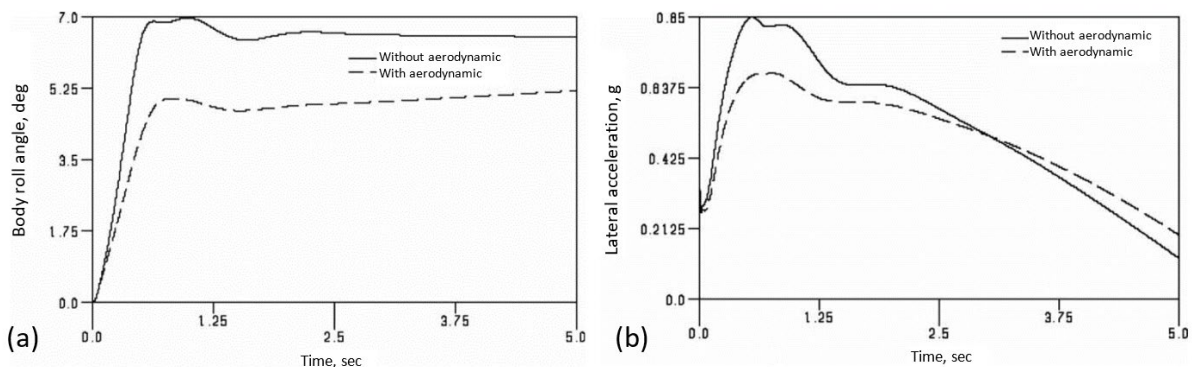

Figure 6: Vehicle lateral motion with and without aerodynamic interactions [91]: (a) Vehicle body roll angle during the cornering manoeuvre, (b) Lateral acceleration during the cornering manoeuvre.

Other additions to multi-body vehicle dynamic analysis have included vehicle occupants in the form of realistic human dummies/humanoids or more detailed biomechanical human skeletal models. These have been used for studies ranging from ride comfort to vehicle impacts. They also aid in the assessment of the performance of vehicle occupant safety features such as seatbelts and airbags by predicting physical injuries to occupants and pedestrians in different vehicle collision scenarios such as frontal, side or rear impacts. Representative literature for multi-body vehicle dynamics, including biomechanical issues are reported in [96-107].

Infinitesimal elasto-multi-body dynamics of powertrain systems

With improved methods of solution, including integration algorithms for stiff systems (widely-split eigenvalue systems or broadband frequency response systems) [14, 24, 108-111], multi-body analysis found its way into the multi-scale domain with integrated simultaneous dynamics across the physics of scale. In this domain, the flexibility of components should be taken into account. Impact dynamics and impulsive responses, typically in vehicular powertrain systems, follow stiff-system characteristics. There has been much research into the inclusion of component flexibility and contact/impact dynamics in multi-body systems. A representative sample [112-117] deals with continuous contact, rigid, elastic and elastoplastic impacts of assembly of parts, with or without friction. There have been occasional reviews of such multi-body analyses [118-121]. Most contact and impact models for multi-body applications have assumed dry rolling conditions [121] or stick-slip conditions [122]. However, in recent years lubricated contacts have also been taken into account.

(i)- Noise, Vibration and Harshness (NVH) of clutches

Applications of multi-body dynamics in stick-slip motion have included the study of untoward NVH phenomena such as judder of dry clutches [122-133]. Clutch judder occurs as the result of stick-slip motion at the clutch friction lining interface, mostly during vehicle take-off (launch). It is the lowest natural frequency (rigid body torsional dynamics mode) of the clutch system which is usually in the

range: 7-10 Hz [24, 127, 134, 135]. Centea et al [124, 126] showed that clutch judder can occur as the result of two causes: (i) - loss of clamp load in hurried release of clutch pedal by the driver (usually within 0.5 s), and/or (ii) - a negative gradient of friction variation with interfacial clutch slip speed.

Figure 7 shows the angular velocities of the engine crankshaft (decreasing through gradual coupling with the drivetrain) and the rising angular velocity of the clutch pressure plate with an initial oscillatory transience. At full engagement, the two velocity variations act in unison. Figure 7(b) shows increased oscillatory behaviour with a negative gradient of friction lining material with increasing slip speed compared with that in figure 7(a), enjoying a positive gradient. The tactile sensations of the driver's foot are more sensitive to an increased oscillatory response. The threshold for tactile perception of pedal oscillations depends on individuals and their footwear. Vehicle occupants notice clutch judder through its coupling dynamics with the fore and aft motion of the vehicle, sometimes described as a kangaroo-type hopping action. This motion is also a function of tyre contact patch longitudinal rolling resistance, the stiffness of the suspension mounts, as well as the stiffness of the tyre belt and carcass. Gradual wear of the clutch friction lining causes a negative gradient, effectively with a reduced coefficient of friction. However, a strong positive gradient can also lead to a rather overdamped clutch response with poor functionality (a "heavy" or "hard" clutch).
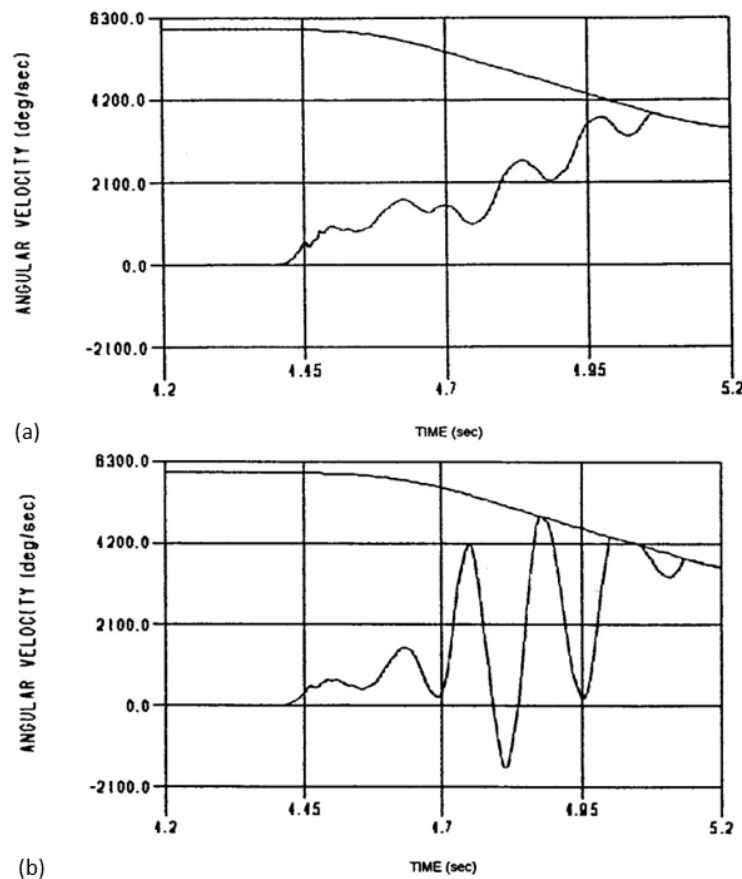

(a)

(b)

Figure 7: Clutch judder with (a) - a positive gradient of clutch friction lining characteristics with slip speed, and (b) - a negative gradient of clutch friction lining characteristics with slip speed

Most clutch lining materials exhibit a negative friction slope with slip speed. Therefore, discerning judder is a function of the severity of the negative slope and the sensitivity of the driver. Loss of clamp load occurs in all forms of manoeuvre, not only due to a hurried release of the clutch pedal. Therefore, for dry clutches, which dominate in the realm of most on-road vehicles, it is essential

to determine the frictional characteristics of the interfacial clutch lining material with slip speed and achieved clamp load. Gkinis et al [136] obtained such characteristics through representative tribometric measurements (figure 8). The line AB shows a typical locus of transient frictional behaviour during clutch engagement reaching a full clamp load (in this case 8900 N for a typical light truck). Note that the characteristics show a negative gradient with increasing slip speed. Gkinis et al [136] used such characteristics in an integrated thermal-dynamic analysis, indicating that even a single clutch actuation can lead to significant interfacial temperature rise. In turn, this rise in temperature significantly affects the path of engagement in the characteristics such as in figure 8. This alters the multi-body dynamics of the clutch mechanism.
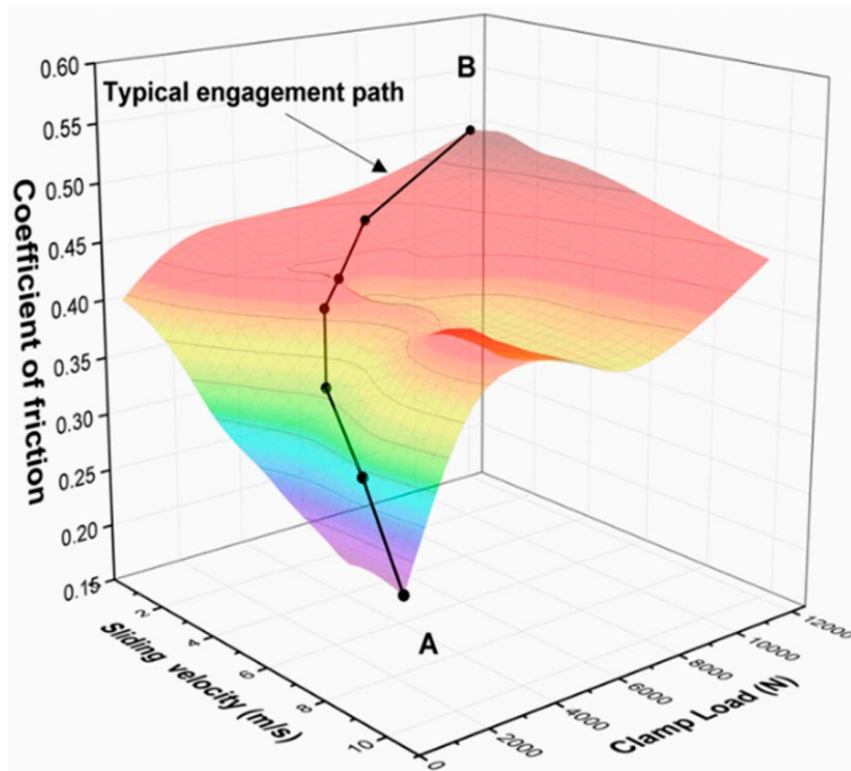

Figure 8: Clutch friction characteristics with slip speed and clamp load [136]

A modern application of the clutch system is to disconnect the engine from parasitic losses which occur in the powertrain system prior to the idling condition. This is an efficiency approach. It means that down-sized engines can be utilised, especially for heavy duty vehicles such as off-road vehicles (loaders, earthmovers and bulldozers). In this configuration, as the engine oil pressure builds up, a hydraulic piston applies a clamp load to the pressure plate and engages the clutch through friction discs (figure 9). Although the hydraulic piston chamber is sealed, engine oil can potentially leak into the frictional interfaces due to seal dynamic failures or wear. This means that a multi-physics multi-body dynamics model should be developed to investigate the effects of fresh or shear-degraded lubricant, which can cause localised stick-slip, on the performance of the clutch system. Dolatabadi et al [137] measured the frictional characteristics of such lubricant-contaminated lining materials and used it in their multi-body dynamic analysis. The torque capacity of the clutch with different oil leakage conditions affects the clutch engagement/disengagement processes, generally reducing the interfacial coefficient of friction, promoting judder. It was shown that full torque capacity is achieved for shear degraded oil, whereas the torque capacity is decreased with any fresh oil leakage (figure 10). The engagement and disengagement initiation times and duration were also delayed due to the presence of lubricant in the contacts.
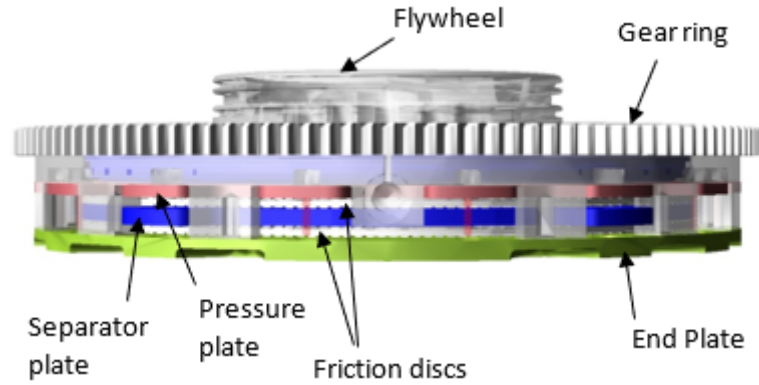
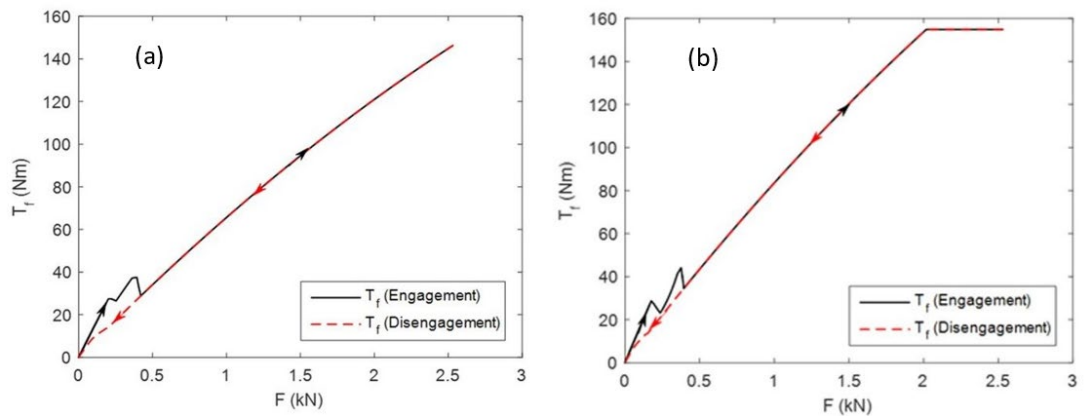Figure 9: Multi-body Dynamic model of the clutch system [137]

Figure 10: Variation of clutch frictional torque, T_f, with the actuation force (clamp load), F, for: (a) fresh-oil leakage and (b) shear-degraded oil leakage [137]

There are other higher frequency NVH issues in clutch systems, including a combined noise and vibration event during clutch pedal movement (during clutch engagement and disengagement). This phenomenon is referred to as in-cycle clutch vibration and is termed in industry as "whoop", which is an onomatopoeic term for the emanated sound from the driver's footwell area [135, 138]. Kelly et al [139] made a multi-body model of a cable-operated clutch system subjected to the motion of the clutch pedal (figure 11). They noted that any potential conical whirl of the flywheel ("flywheel nodding") seen in the figure would impact the clutch friction disc during clutch actuation shown in the monitored vibration transmission path in figure 12.
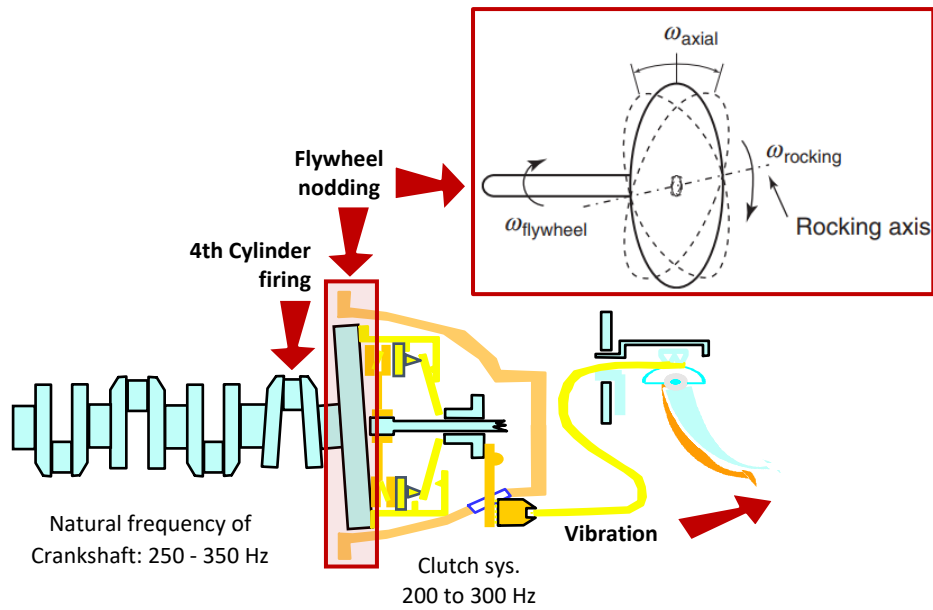
Figure 11: Multi-body model of clutch mechanism during engagement/disengagement
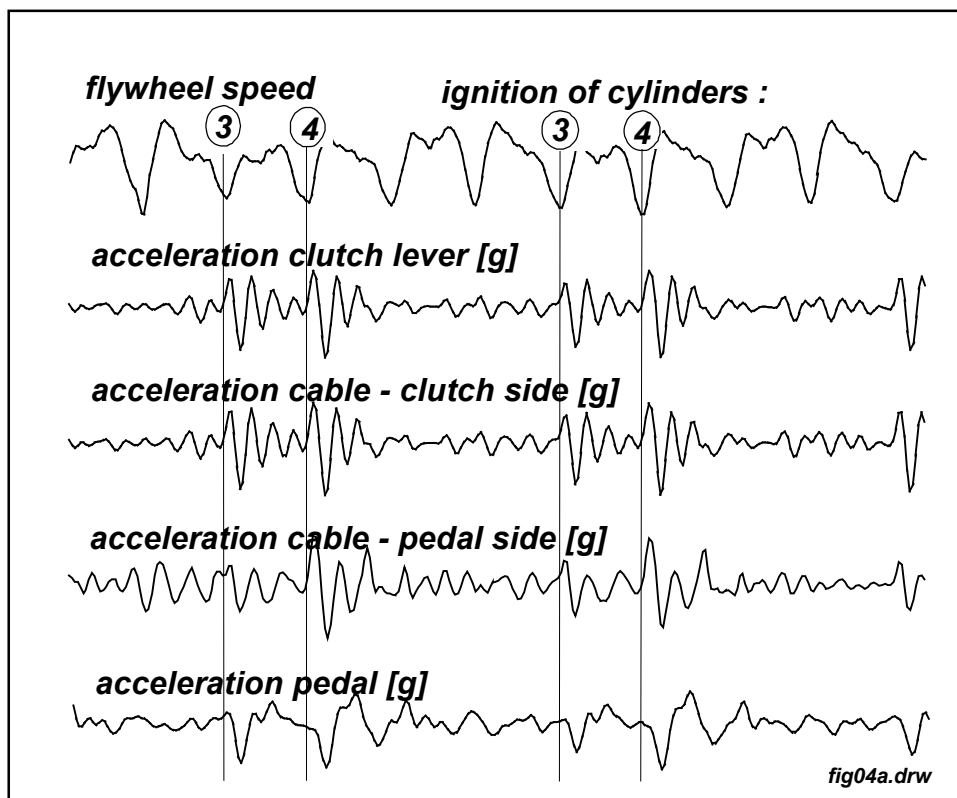
Figure 12: Vibration transmission path from cylinder firing through to clutch components with clutch pedal motion [135, 139]

Clutch whoop coincides with the natural frequency of the clutch lever, which is typically in the range 150-300 Hz for most saloon vehicles that use a cable-operated clutch. Therefore, one method of palliation is to use a mass-damper attached to the clutch lever in order to break the effect of frequency modulation [138, 139].

Figure 12 shows that for a 4-cylinder in-line diesel engine, the firing of cylinders 3 and 4, nearer to the flywheel, accentuate the transmitting vibration signal, which is the underlying cause of clutch whoop. The whoop phenomenon is noted in modern diesel-operated engines with flexible crankshafts. Diesel engines have a higher torque output and with the modern concept of low weight-to-high output power ratio, torsional deflection modes of flexible crankshafts are excited (natural frequencies 250-350 Hz). This leads more readily to the conical whirl of the flywheel (with the nodding motion shown in figure 11 its inset). Therefore, in order to observe the whoop phenomenon in its entirety through numerical analysis, an elasto-multi-body model of the engine is required, coupled with the clutch actuation model. This is described later under Tribo-elasto-multi-body Dynamics of IC Engines.

(ii)- Elasto-multi-body Dynamics of Driveline Systems

Fore and aft motion of the vehicle is not only caused by the clutch judder phenomenon but can also take place through the application of throttle tip-in to accelerate, or throttle back-out to decelerate. These motions can lead to torsional oscillations of the entire driveline system, comprising the clutch, the transmission, the driveshaft, the differential, and the rear axle in the case of rear-wheel-drive (RWD) vehicles. The lowest rigid-body torsional mode of the system is known as shuffle which couples with the fore and aft motion of the vehicle body in its lowest longitudinal vibration mode, known as shunt [123, 140-142]. Shuffle and shunt are usually low frequency harshness-type motions in the frequency range 3-5 Hz for road vehicles and with even lower frequency oscillations (1-3 Hz) for heavy goods vehicles. This depends on vehicle body mass and driveline inertia, stiffness of suspension bushings and tyre-road traction characteristics [142, 143]. The phenomenon affects all vehicles whatever the source of propulsion: IC engines, hybrids, and electrics [144, 145].

Farshidianfar et al [141] and Menday et al [146] used an experimental rig, based on an RWD light truck's driveline (figure 13(a)). The torque lever arm, connected to the flywheel through a low inertia disc brake, is displaced in order to compress the clutch springs and to take-up all the system lashes. The stored energy is then released within 80-100 ms in the form of an impulse torque of broad-band frequency composition. This action excites a broad-band response of the drivetrain system, from low frequency rigid body shuffle oscillations of the driveline to high frequency elasto-acoustic response of modern hollow (2 mm wall thickness) driveshaft pieces. Figure 13(b) shows the position of accelerometers and microphones placed for the acquisition of NVH signals.
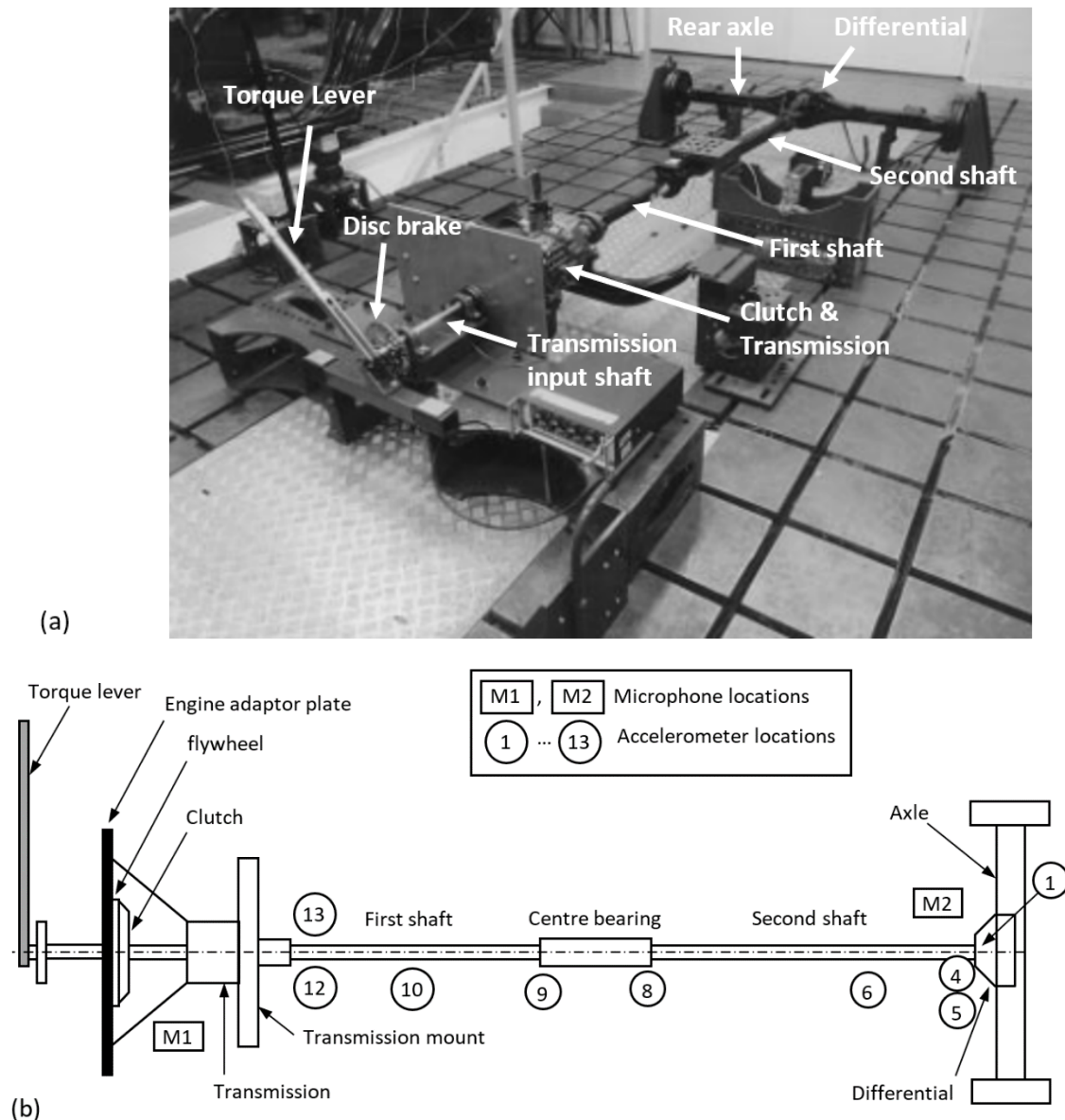
Figure 13: (a) The powertrain rig, (b) schematic representation of the rig [141, 146]

Figure 14(a) shows a model of the powertrain system by Farshidianfar et al [141], primarily for the study of shuffle. Figure 14(b) shows a typical signal comprising a series of high frequency oscillations occurring in a cyclical manner [147]. These high frequency oscillations are the result of impacts in the lash zones (such as in the clutch and the transmission) with cyclic overall oscillations of rigid body driveline (shuffle). An overall Fast Fourier Transformation (FFT) of the cyclic oscillations leads to the identification of shuffle response at around 3 Hz [141]. Figure 15 shows the contents of a window around one of the regions of high frequency oscillations [146, 148]. These are the result of hard impacts in the lash zones, particularly the transmission gearing pairs (1-2 ms), first reported by Krenz [149]. The applied torque (in this case 150 Nm) is achieved through the movement of the torque lever arm (figure 13(a)). With release of the lever the released energy travels along the driveline, inducing elasto-acoustic responses of thin-walled driveshaft pieces (tubes). Owing to the thinness of the tubes, they possess a high modal density (broad-band elastic modal responses), typically 500-5000 Hz [146-149]. This elasto-acoustic phenomenon is referred to as clonk or clunk in industry (another onomatopoeic term as in the case of clutch whoop). It is particularly troublesome in trucks, buses, coaches and lorries with abrupt throttle tip-in or back-out or with sudden release of clutch [146, 150,

151]. It also depends on the laden state of the vehicle (Figure 16) [152]. The problem of clonk persists to a certain extent even with the use of dual-clutch transmissions to avoid sudden release [153, 154]. Although no component failure actually occurs with clonk, the metallic high frequency sharp acoustic emission is very disconcerting to the driver, vehicle occupants and even to other road users.
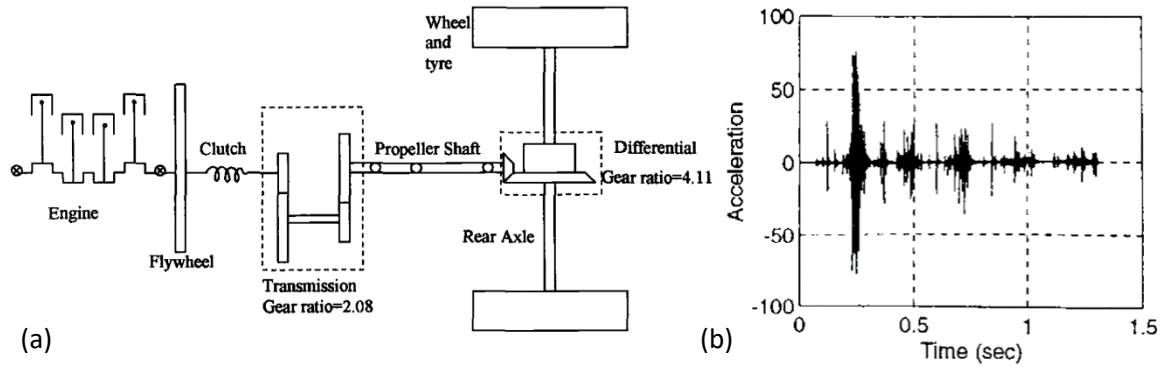

(a)

(b)

Figure 14: (a) Schematic representation of the multi-body model [141], and (b) Clonk response in cycles of shuffle [147]
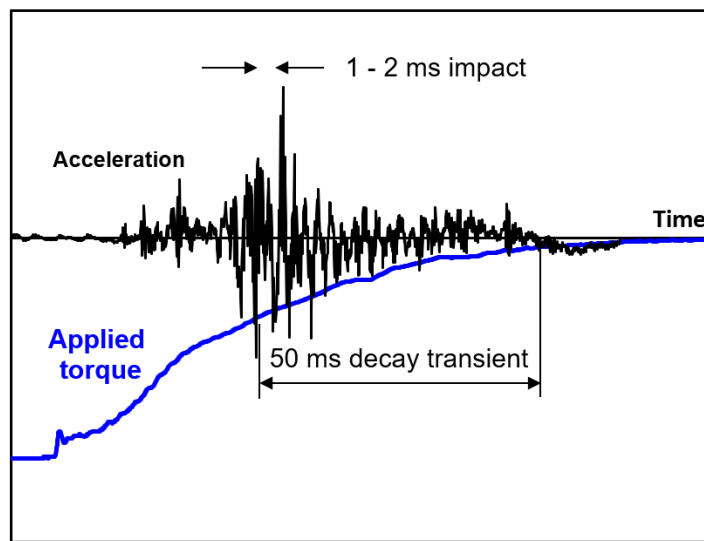

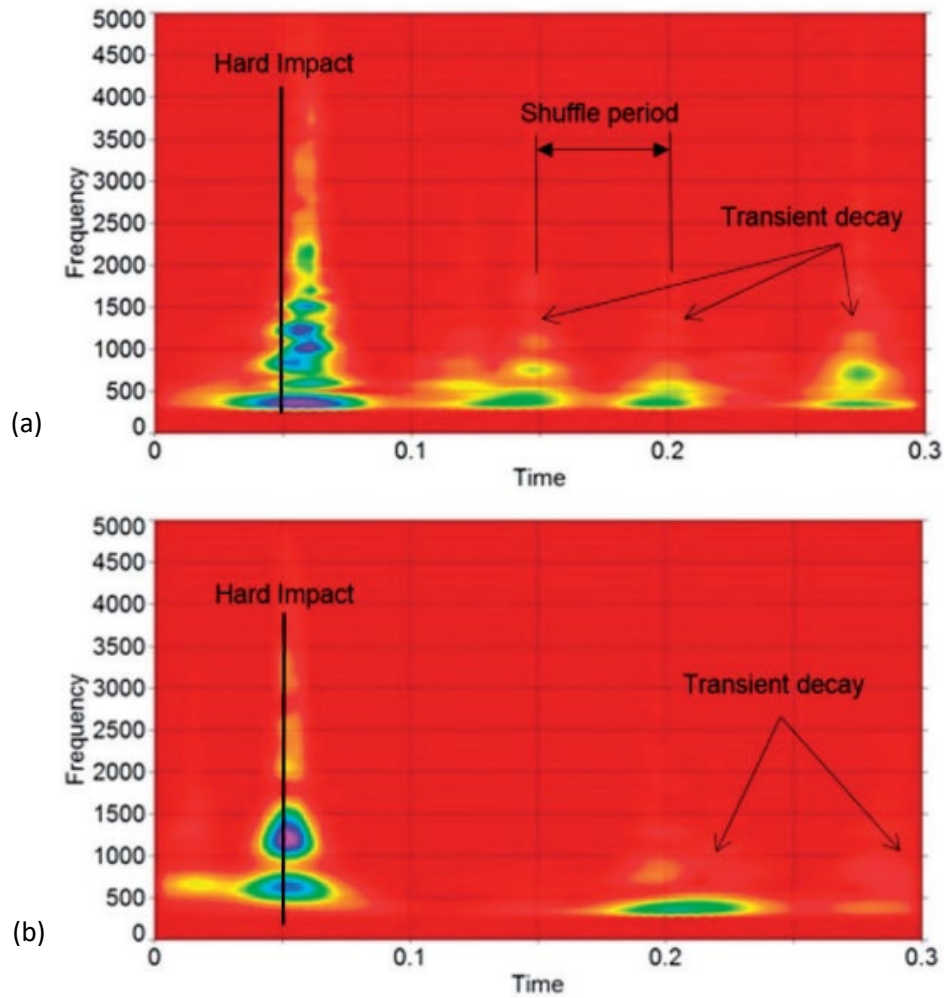Figure 15: A typical clonk signal (146-148]

Figure 16: Wavelet spectrum of predicted elasto-acoustic response of vehicular driveline system from hard impulsive impact to transient decay (a) for an unladen truck and (b) for a half-laden truck [152]

The shuffle response predictions of the reported model of Farshidianfar et al [141] agree well with the spectral analysis of the overall signal in Figure 14(b) [147]. This shows that a simple rigid multi-body analysis can be used effectively to predict some powertrain NVH phenomena (Figure 17). As can be observed, the shuffle frequency for this RWD truck is measured and predicted to be at approximately 3 Hz. However, the signal in figure 14 (b) contains repetitive high frequency oscillations with a very short duration hard impact, followed by a transient decay (requiring windowing techniques to determine the high frequency content). To predict the high frequency elasto-acoustic clonk signal content, flexible multi-body dynamics analysis needs to be carried out. Such models contain modal composition of the hollow thin driveshaft tubes. A flexible multi-body dynamics model for the driveline system was first developed by Arrundale et al [155]. Figure 18 shows the driveline response when subjected to abrupt application of torque as in the sudden release of clutch pedal. The cycles of shuffle can be noted, with clonk signal with every cycle of shuffle, similar to the measurements in Figure 14(b). In the case of predictions there is no cyclic attenuation of the clonk events. This is because the model does not include material hysteretic damping or dry friction in joints and splines, which are present in practice. Nevertheless, the suitability of flexible multi-body analysis to predict high frequency NVH events was then established by the contribution of Arrundale et al [155].
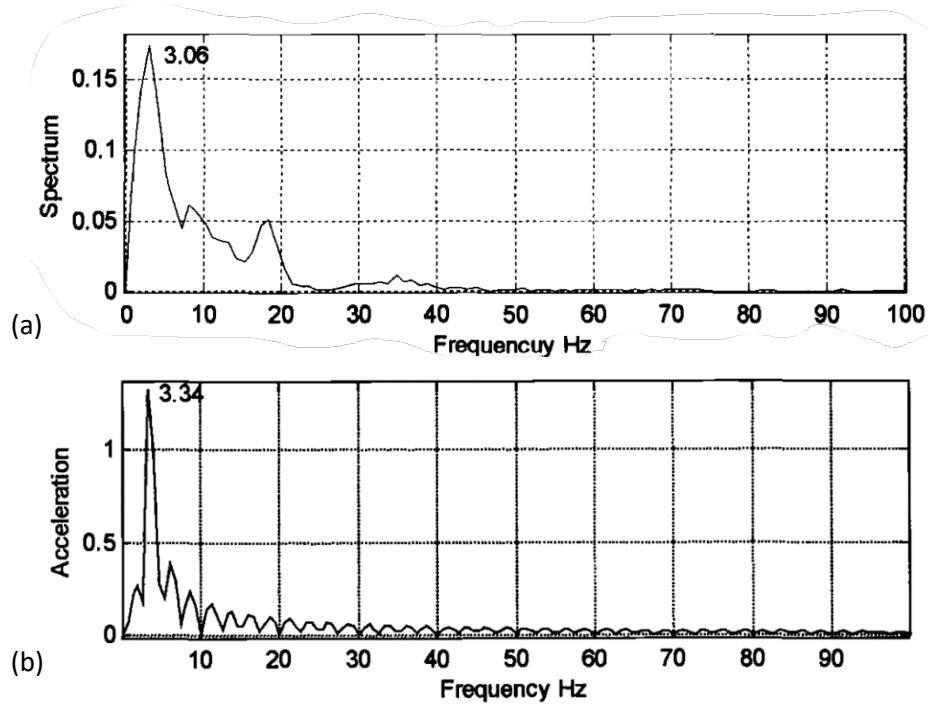
Figure 17: (a) Typical spectrum of the measured output response at transmission [147], and (b) predicted spectral content of the shuffle response [141]
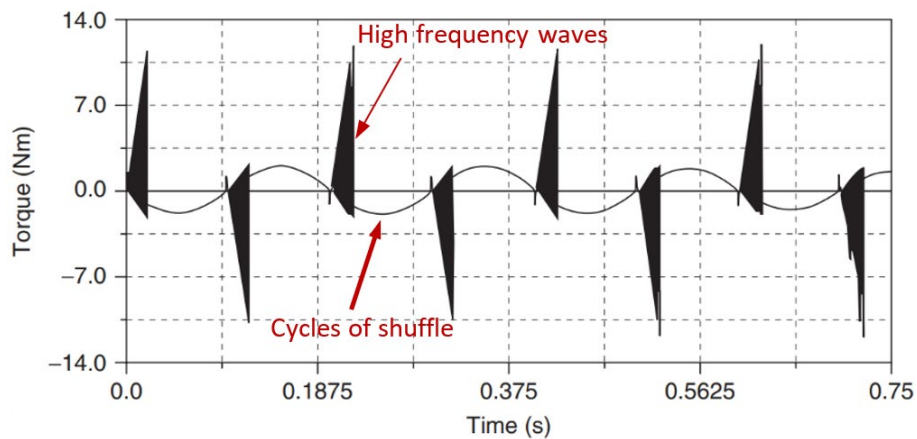
Figure 18: Cycles of shuffle with high frequency impulsive action through zones of low lash rate zones (clonk) [155]

Of course, the main purpose of any NVH analysis is to determine the underlying cause of any manifested phenomenon. This is followed by scenario-building simulations to achieve palliation or overall system refinement. Detailed elasto-multi-body dynamic analysis of vehicular powertrain systems under the clutch-induced clonk condition was carried out by Theodossiades et al [156], who used component mode synthesis [157, 158] to obtain the modal characteristics of the hollow driveshaft tubes.

Some predicted modal behaviours of the driveline system are shown in figure 19. The excited mode shapes comprise relatively low frequency bending modes through to torsional and combined torsional-deflection (flexural) modes. Some of the high frequency flexural modes are efficient noise radiators as they coincide with the acoustic modes of the hollow cavities of the driveshaft tubes. These modes are referred to as the breathing (or bellowing) modes and are shown in figure 19. Therefore,

methods of palliation such as foam filling the tubes, or fitting them with cardboard liners/inserts can be used to break the propagation of high frequency waves.

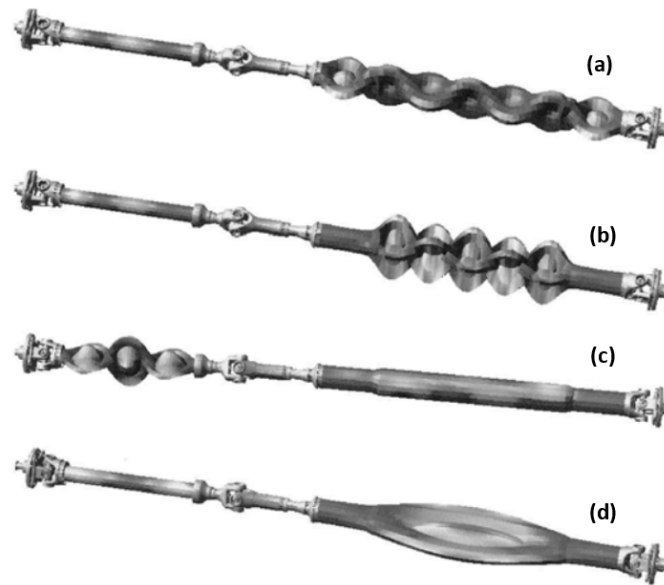Figure 19: Magnified graphical scale of dominating mode shapes corresponding to (a) 3853 Hz, breathing mode, (b) 3550 Hz, breathing mode, (c) 3254 Hz, breathing mode, and (d) 617 Hz, breathing mode [156]

It is important point to note that as long as lash zones, such as clutch springs, meshing of transmission and differential gears, driveline splines and others exist, an impulsive action can result in the take-up of lash. Therefore, the abrupt release of clutch, sudden gear shift, or throttle tip-in or back-out can lead to elastic wave propagation. Meshing of transmission gears can initiate other NVH phenomena of concern in industry, such as whine of gears, commonly referred to as axle whine [159-162], gear rattle under various loading conditions, including engine idling, creep and drive rattle [163-167]. Investigations regarding these phenomena show that in the study of impact problems through lash zones, it is essential to include the action of lubricant squeeze film motion as well as friction. Therefore, multi-body analysis should take into account inertial dynamics, structural elastodynamics, contact/impact mechanics, as well as other physics such as thermodynamics. As the multi-body investigations gradually extended into the area of powertrain NVH, analyses tended towards multi-physics multi-scale approach.

Multi-physics multi-scale extensions of multi-body dynamics

The first substantive application of multi-body analysis in vehicular powertrain systems was the multi-physics of the internal combustion (IC) engine or its sub-systems. The investigation usually comprised inertial dynamics of crank-connecting rod-piston assembly with the inclusion of measured combustion variation with engine speed. The analysis also needs to incorporate lubricant reaction and friction of load bearing conjunctions such as the main crankshaft support bearings and piston-cylinder surface interactions. The first reported analyses addressed the more confined sub-systems such as tribo-dynamics of piston systems [168-171] and valvetrains [172-177], some using Newton-Euler approach and some using constrained Lagrangian dynamics. The key point is that applications of multi-body analysis extended from large displacement low frequency vehicle dynamics to infinitesimal-displacement-high frequency domains. There have been many refined analyses for both piston-

cylinder systems and valvetrains since early tribo-dynamic analyses. Recalling them all is outside the scope of this contribution. Instead, attention here is directed towards multi-physics of overall IC engines.

(i)- Tribo-elasto-multi-body dynamics of IC engines

During the 1970s-80s, the main focus of multi-body analysis for vehicle engineering applications was large displacement dynamics problems, such as suspension analysis, vehicle ride and handling manoeuvres. These issues have been addressed in the previous sections. With the advent of software codes such as ADAMS [13] and DADS, multi-body analysis became an almost routine exercise in the automotive industry. In the 1980s and 90s, improved integration techniques [14] for high frequency NVH events emanating from stiff-system characteristics enabled multi-body dynamics to enter into the area of powertrain engineering. One of the very first applications was the complex issue of dynamics of IC engines.

Unlike the standard crank-connecting rod-piston assembly that is often used as a tutorial example, IC engines comprise important components such as crankshaft support bearings, big-end and small-end bearings. These bearings have often been treated incorrectly as mere joints (simply as holonomic or non-holonomic algebraic constraints). Inclusion of lubricated bearings and friction of interacting surfaces such as piston-cylinder leads to multi-physics analysis as described above. The system comprises different scales of interactions/motions from microscale in tribological conjunctions to infinitesimal vibration of flexible components such as crankpins and webs, and onto large displacements of piston, crankshaft/flywheel. This is truly a multi-physics multi-scale problem. Some of the first reported engine dynamics models [178-182] have included at least some of these features, as well as incorporating measured combustion characteristics, which contains a broadband signature of the combustion process itself, depending on the 2- or 4-stroke process [24].

A multi-body model of a single cylinder engine was developed by Boysal and Rahnejat [179, 180] in ADAMS. This was a detailed model that included hydrodynamics of the crankshaft support bearing. It also included hydrodynamic interactions between the piston compression ring and the cylinder wall. Therefore, a multi-scale tribo-dynamics model was created that also included thermodynamic solution of the combustion process. The instantaneous combustion pressure was obtained through the application of the first law of thermodynamics to the trapped air-fuel mixture in the combustion chamber. The Wiebe functions were used to determine the fraction of burned fuel, and Woschini correlation was employed to determine the instantaneous heat transfer coefficient. This was the first detailed MBD engine model. However, it did not include the effect of flexibility of inertial components. Torsional vibrations of the engine were obtained including the effect of 4-stroke combustion signature, rigid body inertial imbalance and synchronous and asynchronous vibrations of engine bearings. The predicted spectrum agreed well with the spectral composition of a single cylinder engine, determined analytically or through the crank-star method established in [24]. The models in [179, 180] did not include the engine valvetrain system or the flexibility of the engine block and the crankshaft. These were included by Zeischka et al [182]. The valve train system was treated as a rigid cam-tappet contacting pair. Other early contributions considered the effect of component flexibility, such as an elastic camshaft [177] and lubricated cam-tappet contact [176, 177]. Ma and Perkins [183] developed an engine model with a flexible crankshaft-connecting rod-piston assembly. They also included a balancer shaft and engine support bearings. They compared their recursive-formulated integrated engine model with an ADAMS model of the same configuration and found reasonable agreement under firing conditions.

The key modelling aspect for modern engines has been the inclusion of crankshaft flexibility. This is because in the past 2-3 decades, with the adoption of the concept of high output power-to-lightweight ratio, a plethora of engine vibration phenomena have emerged. The vibration response of modern engines is now strewn with spectral contributions which had been palliated previously by cylinder phasing in order to reduce inertial imbalances and fluctuations in the combustion signature [24]. However, lightweight crankshafts made of materials of lower elastic moduli with thinner cross-sections or hollow structures are subject to combined torsional-deflection modes. Thus, the vibration spectrum of such engines contains half engine order multiples that do not exist in balanced rigid body rotation of stubby crankshafts [24].

Like many other structures, crankshaft flexibility was initially achieved by flexible crank-pins, discretised by a series of Eulerian beams. This approach is known as Transfer Matrix Method (TMM). An extended approach is to structure the crankshaft as a series of interconnected beams in 3-dimensions, represented by Dynamic Stiffness Matrix Method (DSMM) [24, 184, 185]. An initial contribution using TMM for a 6-cylinder inline engine was reported by Arrundale et al [186], taking into account the flexible crankshaft torsional deflection modes at half engine orders. These spectral contributions are termed engine roughness. The work of Arrundale was followed by Ma and Perkins [183].
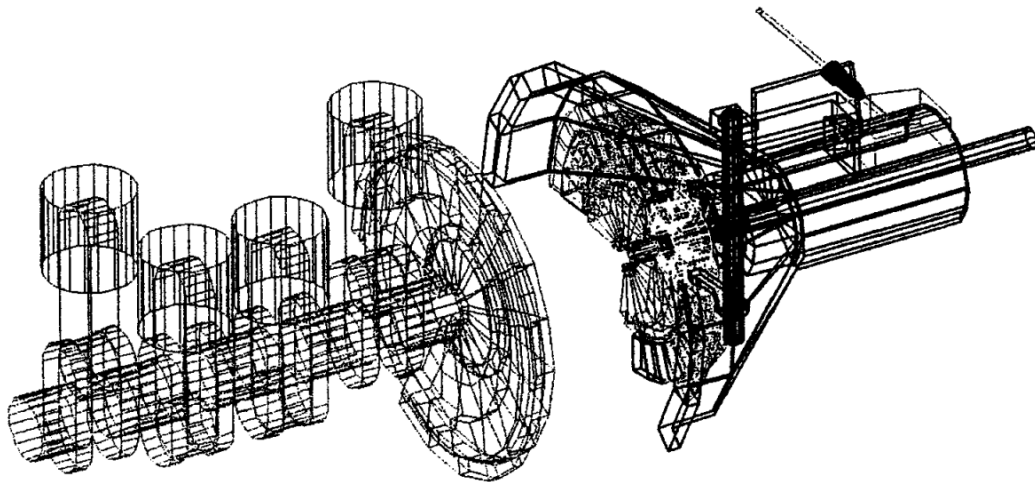

Figure 20: Combined multi-body engine and clutch system (integrated at the flywheel position) [187]

Kushwaha et al [187] modelled a 4-stroke 4-cylinder engine, coupled to a clutch model with a flexible clutch-cover in order to study the clutch whoop phenomenon described above (figure 20). They studied the engine vibration signals, both with a rigid crankshaft and a flexible crankshaft, with an appropriate cylinder phasing and firing order. For the rigid crankshaft, the spectrum of vibration at the position of the flywheel comprises only the second engine order and its higher harmonics as would be expected of a 4-cylinder 4-stroke engine [24] (figure 21). For a realistic flexible crankshaft, all engine order harmonics, including half-engine orders excited by the 4-stroke combustion signature, are present (figure 22). Therefore, it was shown that engine roughness as a source of NVH in modern vehicles is the penalty paid for improved fuel efficiency under the modern trend of high output power-to-lightweight ratio. The problem of exacerbated NVH with reduced inertial dynamics in favour of

elastodynamics of flexible parts is likely to continue into the future with alternative sources of propulsion such as hybrid engines, electric vehicles and dual-fuel engines.
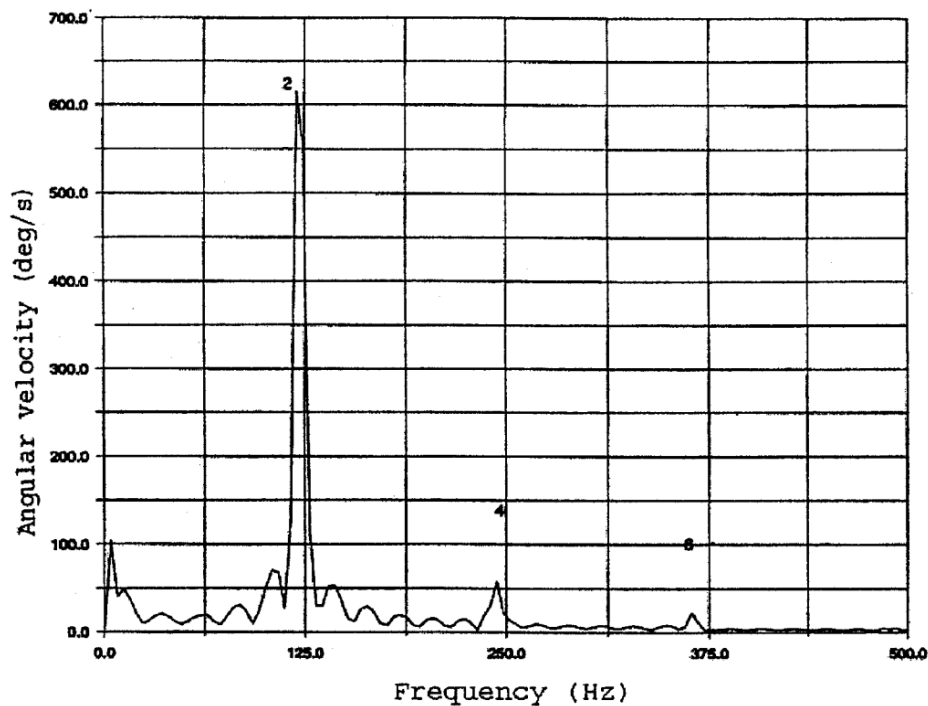

Figure 21: Spectrum of vibration of flywheel motion of a 4-cylinder 4-stroke inline multi-body engine model with a rigid crankshaft [187]

Like the inclusion of component flexibility, friction generated by surface interactions significantly affects the inertial dynamics of engines. Guzzomi et al [188] took into account the generated friction between the piston and cylinder surface. They showed that the accuracy of their predictions improved markedly when compared with the experimental measurements of engine block dynamics by Watling [189] using a devised test rig. Without friction in the analysis, the accuracy of predictions was compromised. Their study showed the importance of tribo-dynamics in engine applications. Perera et al [190, 191] developed a tribo-elasto-multi-body dynamic model, which included the flexibility of crankshaft as well as the tribology of piston-cylinder conjunctions (piston skirt and compression ring). They also included the main crankshaft support bearing. The authors carried out component mode synthesis [157, 158, 192] to include the structural dynamics of flexible components such as the crankshaft and the flywheel.

Figure 23 shows the model of a single cylinder Ricardo E6 engine developed in [190, 191]. An interesting study carried out in [191] concerns the relatively recent introduction of crankshaft offset instead of the traditional piston offset. Both offsetting methods are used to eliminate/reduce the secondary motions of the piston, which often lead to piston slap near the top dead centre. Secondary piston motions refer to its lateral excursions and tilting motion about its lateral axis within the confine of the cylinder. Figure 24 shows that, with relatively small crankshaft axis offset, there is reduced frictional power loss in the thrust and anti-thrust sides during parts of the piston cycle subjected to secondary inertial dynamics.
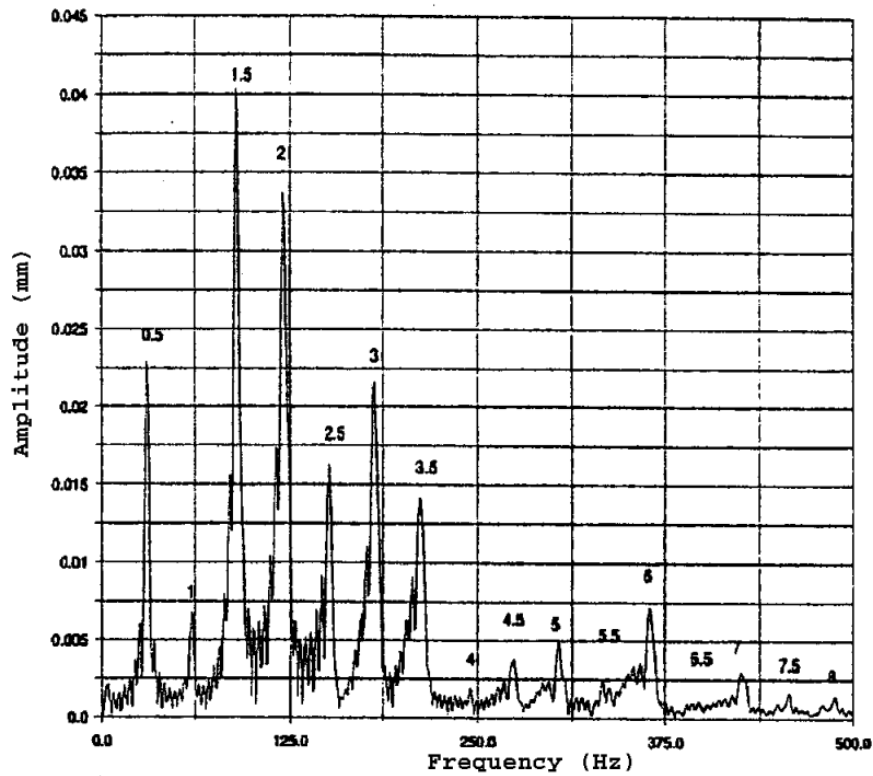
Figure 22: Spectrum of vibration of flywheel conical motion of a 4-cylinder 4-stroke inline multi-body engine model with elastic crankshaft torsional deflection [187]
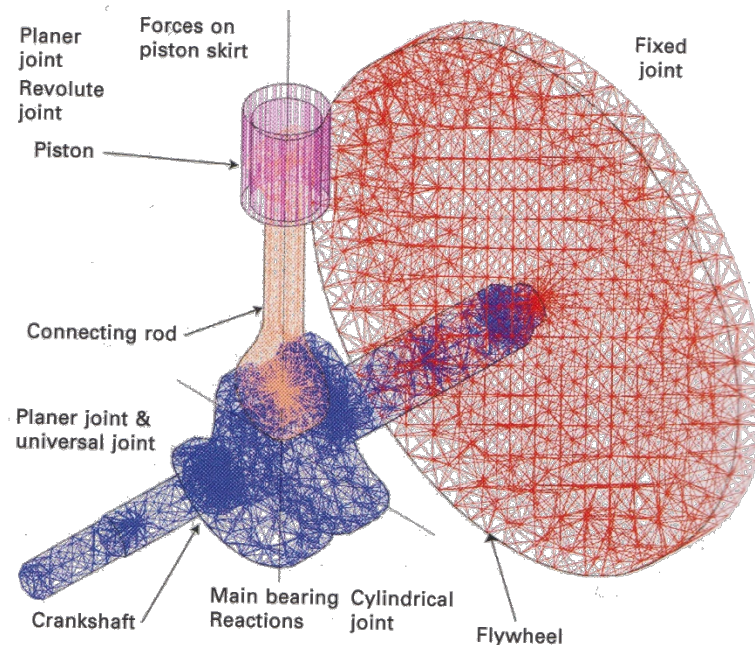
Figure 23: Tribo-elasto-multi-body dynamics model of a single cylinder engine [190, 191]

Another integrated multi-body approach taking into account inertial dynamics, thermodynamics and friction as well as stiffness and damping torque was reported by Giakoumis et al [193]. They also showed that crankshaft stiffness and engine loading, depending on combustion and engine aspiration, plays a significant role in system dynamics. Their predictions agreed well with their

experimental investigations of a single cylinder engine. Another engine dynamics model was presented by Karabulut [194] for the case of a 2-cylinder 4-stroke engine, including the engine block and mounts. The model studied the detailed design of engine components and their effects on the generated inertial forces transmitted to the vehicle chassis.
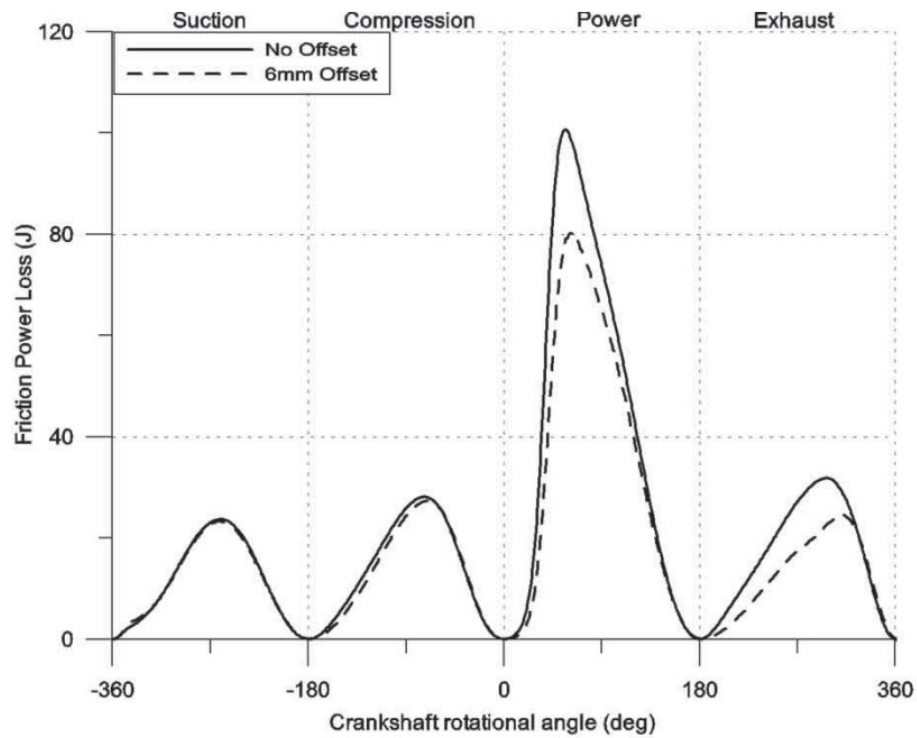
(a) Thrust side
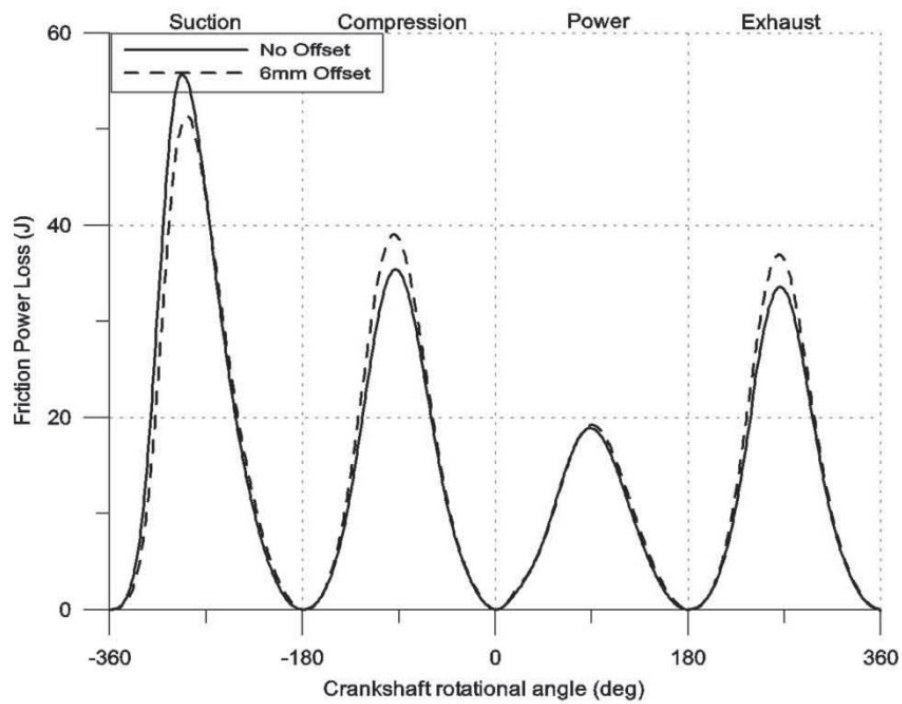
(b) Anti thrust side

Figure 24: Frictional power loss at piston skirt to cylinder wall contacts with and without crankshaft offset [191]

MBD of IC engines continues to attract attention even though there is a growing downturn in their future outlook. The current trend is to model vehicular propulsion in the context of hybrid electric-IC engines, as well as smaller engines acting as range extenders (such as single, twin or 3-cylinder configurations) [195-197]. With the emergence of dual-fuel engines, such as hydrogen-diesel, particularly with potential use in marine and heavy vehicle applications, multi-body analysis will continue to play a role in the prediction of performance of various propulsion systems. Some recent analyses include [198-200].

Concluding Remarks

The foregoing shows that MBD analysis entered the application domain with four main developments: (i)- automatic generation of equations of motion, (ii)- solution of generated DAEs, (iii)- sparse matrix technology and (iv)- improved step-by-step integration methods. These developments led to MBD analysis codes such as ADAMS and DADS, which were initially used by the automotive and aerospace industries. This paper covers applications of multi-body dynamics in vehicle engineering.

In vehicle engineering, the original widespread use of MBD has been in relatively large displacement dynamics (suspension analysis, ride comfort assessment and vehicle handling/road holding). Increasingly, the role of component flexibility in MBD analyses has become commonplace (elasto-kinetics of suspensions, impacts, etc). Therefore, elasto-multi-body dynamics has embraced component flexibility with resulting infinitesimal vibrations (modern MBD analysis is carried out in a multi-scale manner). Furthermore, the use of MBD in powertrain analysis has resulted in the inclusion of other physical phenomena such as contact/impact dynamics, friction in microscale and generated heat. Thus, the state of the art in modern multi-body analysis has become multi-physical and multi-scale. Over the past 25 years, the Journal of Multi-body Dynamics (JMBD) has been at the forefront of many developments, some of which have been recalled and expanded upon in this paper.

Acknowledgements

References

[1]   Da Vinci, L., "Manuscript B", Institut de France, Paris, 1487-1490.

[2]   Galileo, G., "Discourses Concerning Two New Sciences", 1638, Leiden, The Netherlands

[3]   Galileo, G., "Dialogue on the Two Chief World Systems", 1632, Florence, Italy

[4]   Kepler, J., "Astronomia Nova", 1609, Graz, Austria.

[5]   Tusi, N., "Zij-i Ilkhani", Maragheh, 1272, Iran (see British Museum, MS Or.7464)

[6]   Copernicus, N, "De revolutionibus orbium coelestium", Johannes Petreius, Nuremberg, 1543

[7]   Newton, I., "Philosophiæ naturalis principia mathematica", Royal Society, London, 1687.

[8]   Euler, L., "Nova methods motum corporum rigidarum determinandi", Novi Commentarii Academiae Scientiarum Petropolitanae, 1776, 20, pp. 208-238.

[9]   Lagrange, J. L., Mecanique Analytique, L'Academie Royal des Sciences, 1788, Paris.

[10]   Gammel, R., "Der Kreisel Seine Theorie und seine Anwendungen", 1st edition, 1920, (Vieweg, Braunschweig); 2nd edition, 1950 (Springer-Verlag, Berlin).

[11]   Magnus, K., "Kreisel-Theorie und Anwendungen", 1971, Springer-Verlag, Berlin.

[12]   Denavit, J. and Hartenberg, R. S., "A kinematic motion for lower pair mechanisms based on matrices", J. Appl. Mech., 1955, 22, pp. 215-221.

[13]   Orlandea, N., Chace, M. A. and Calahan, D. A., "A sparsity-oriented approach to the dynamic analysis and design of mechanical systems, Parts I and II", Trans. ASME, J. Engng. for Industry, 1977, 99, pp. 773-784.

[14]   Gear, C. W., "Simultaneous numerical solution of differential-algebraic equations", Circuit Theory, 1971, 18, pp. 89-95.

[15]   Segel, L., "Theoretical prediction and experimental substantiation of the response of the automobile to steering control, in research in automobile stability and control and in tyre performance", Proc. IMechE, 1956-7 (7), pp. 310-330.

[16]   McHenry, R. R., "An analysis of the dynamics of automobiles during simultaneous cornering and ride motions, in handling of vehicles under emergency conditions", Proc. IMechE, 1968-9, 13, pp. 28-48.

[17]   Kortum, W. and Sharp, R. S.,"Multi-body Computer Codes in Vehicle System Dynamics", Swets und Zeitlinger, Amsterdam, 1973.

[18]   Kortum, W. and Schiehlen, W., "General purpose vehicle system dynamics software based on multi-body formalisms", Veh. Sys. Dynamics, 1985, 14, pp. 229-263.

[19]   Sharp, R. S., "The application of multi-body computer codes to road vehicle dynamics modelling problems", Proc. IMechE, Part D: Journal of Automobile Engineering, 1994, 208(D1), pp. 63-70.

[20]   Sharp, R. S. "C553/045/98: Multi-body dynamics applications in vehicle engineering", IMechE Conference Transactions, 13, Mechanical Engineering Publications, 1998.

[21]   Dugoff, H., Fancher, P.S. and Segel, L., "An analysis of tire traction properties and their influence on vehicle dynamic performance", SAE Trans., 1970, pp. 1219-1243.

[22]   Bakker, E., Nyborg, L. and Pacejka, H.B., "Tyre modelling for use in vehicle dynamics studies", SAE Trans., 1987, pp. 190-204.

[23]   Pacejka, H.B., "The magic formula tyre model", Vehicle System Dynamics, 1992, 21(S1), pp. 1-18.

[24]   Rahnejat, H., "Multi-body Dynamics: Vehicles, Machines and Mechanisms", Professional Engineering Publishing, Bury St Edmunds, UK, 1998.

[25]   Blundell, M.V., "The modelling and simulation of vehicle handling Part 2: vehicle modelling", Proc. IMechE, Part K: J. Multi-body Dynamics, 1999, 213(2), pp. 119-134.

[26] Hegazy, S., Rahnejat, H. and Hussain, K., "Dynamic tyre testing for vehicle handling studies", Multibody Dynamics: Monitoring and Simulation Techniques II, 2000, pp. 135-154.

[27] Hegazy, S., Rahnejat, H. and Hussain, K., "Multi-body dynamics in full-vehicle handling analysis", Proc. IMechE, Part K: J. Multi-body Dynamics, 1999, 213(1), pp. 19-31.

[28] Blundell, M.V., "The modelling and simulation of vehicle handling Part 4: Handling simulation", Proc. IMechE, Part K: Journal of Multi-body Dynamics, 2000, 214(2), pp. 71-94.

[29] Hegazy, S., Rahnejat, H. and Hussain, K., "Multi-body dynamics in full-vehicle handling analysis under transient manoeuvre", Vehicle System Dynamics, 2000, 34(1), pp. 1-24.

[30] British Standard Automobile Series, "BS AU 230: Lateral transient behaviour of passenger cars", 1989.

[31] Abe, M., "Vehicle dynamic and control for improving handling and active safety: from 4WS to DYC", In Proceedings of IMechE Conference on Multi-body Dynamics: New Techniques and Applications, London, 1998, paper C553/047/98, pp. 229-248.

[32] Abe, M., "Vehicle dynamics and control for improving handling and active safety: from four-wheel steering to direct yaw moment control", Proc. IMechE, Part K: J. Multi-body Dynamics, 1999, 213(2), pp. 87-101.

[33] Matschinsky, W., "Road vehicle suspensions", Professional Engineering Publishing, Bury St Edmunds, UK, 2000.

[34] Gillespie, T., "Fundamentals of vehicle dynamics", SAE International, Warrendale, PA, USA, 1992.

[35] Crolla, D.A., "Vehicle dynamics—theory into practice", Proc. IMechE, Part D: J. Automobile Engineering, 1996, 210(2), pp. 83-94.

[36] Pesterev, A.V., Bergman, L.A. and Tan, C.A., "A novel approach to the calculation of pothole-induced contact forces in MDOF vehicle models", J. Sound and Vibration, 2004, 275(1-2), pp.127-149.

[37] Azman, M., Rahnejat, H., King, P.D. and Gordon, T.J., "Influence of anti-dive and anti-squat geometry in combined vehicle bounce and pitch dynamics", Proc. IMechE, Part K: J. Multi-body Dynamics, 2004, 218(4), pp. 231-242.

[38] Pesterev, A.V., Bergman, L.A., Tan, C.A. and Yang, B., "Assessing tire forces due to roadway unevenness by the pothole dynamic amplification factor method", J. Sound and Vibration, 2005, 279(3-5), pp. 817-841.

[39] Azman, M., King, P.D. and Rahnejat, H., "Combined bounce, pitch, and roll dynamics of vehicles negotiating single speed bump events", Proc. IMechE, Part K: J. Multi-body Dynamics, 2007, 221(1), pp. 33-40.

[40] von Chappuis, H., Mavros, G., King, P.D. and Rahnejat, H., "Prediction of impulsive vehicle tyre-suspension response to abusive drive-over-kerb manoeuvres", Proc. IMechE, Part K: J. Multi-body Dynamics, 2013, 227(2), pp.133-149.

[41] Sharp, R. S., "Influences of suspension kinematics on pitching dynamics of cars in longitudinal maneuvering", Vehicle System Dynamics, 1999, 33, pp. 23–36.

[42] Sun, T.C., Luo, A.C. and Hamidzadeh, H.R., "Dynamic response and optimization for suspension systems with non-linear viscous damping", Proc. IMechE, Part K: J. Multi-body Dynamics, 2000, 214(3), pp. 181-187.

[43] Hamed, M., Tesfa, B., Gu, F. and Ball, A.D., "Vehicle suspension performance analysis based on full vehicle model for condition monitoring development", Vibration Engineering and Technology of Machinery, 2015, pp. 495-505.

[44] Cao, D., Song, X. and Ahmadian, M., "Editors' perspectives: road vehicle suspension design, dynamics, and control", Vehicle System Dynamics, 2011, 49(1-2), pp. 3-28.

[45] Čorić, M., Deur, J., Xu, L., Tseng, HE. and Hrovat, D., "Optimisation of active suspension control inputs for improved vehicle ride performance", Vehicle System Dynamics, 2016, 54(7), pp. 1004-1030.

[46] Qin, Y., Wang, Z., Xiang, C., Hashemi, E., Khajepour, A. and Huang, Y., "Speed independent road classification strategy based on vehicle response: Theory and experimental validation", Mechanical Systems and Signal Processing, 2019, 117, pp. 653-666.

[47] Pacejka, H.B. and Sharp, R.S., "Shear force development by pneumatic tyres in steady state conditions: a review of modelling aspects", Vehicle System Dynamics, 1991, 20(3-4), pp. 121-175.

[48] Hegazy, S., Rahnejat, H. and Hussain, K., "Dynamic tyre testing for vehicle handling studies", Multibody Dynamics Monitoring and Simulation Techniques II, 2000, pp. 135-154.

[49] Lugner, P., Pacejka, H.B. and Plöchl, M., "Recent advances in tyre models and testing procedures", Vehicle System Dynamics, 2005, 43(6-7), pp. 413-426.

[50] Pacejka, H.B., "Tire and vehicle dynamics", Elsevier, Amsterdam, 2005.

[51] Li, B., Yang, X. and Yang, J., "Tire model application and parameter identification-a literature review", SAE International Journal of Passenger Cars-Mechanical Systems, 2014, 7(2014-01-0872), pp. 231-243.

[52] Singh, K.B., Arat, M.A. and Taheri, S., "Literature review and fundamental approaches for vehicle and tire state estimation", Vehicle System Dynamics, 2019, 57(11), pp. 1643-1665.

[53] Zhou, J., Wong, J.Y. and Sharp, R.S., "A multi-spoke, three plane tyre model for simulation of transient behaviour", Vehicle System Dynamics, 1999, 31, pp. 35–45.

[54] Deur, J., Asgari, J. and Hrovat, D., "A 3D brush-type dynamic tire friction model", Vehicle System Dynamics. 2004, 42(3), pp. 133-173.

[55] Mavros, G., Rahnejat, H. and King, P.D., "Transient analysis of tyre friction generation using a brush model with interconnected viscoelastic bristles", Proc. IMechE, Part K: J. Multi-body Dynamics, 2005, 219(3), pp. 275-283.

[56] Svendenius, J., Gäfvert, M., Bruzelius, F. and Hultén, J., "Experimental validation of the brush tire model", Tire Science and Technology, 2009, 37(2), pp.122-137.

[57] Mavros, G., Rahnejat, H. and King, P., "Analysis of the transient handling properties of a tyre, based on the coupling of a flexible carcass—belt model with a separate tread incorporating

transient viscoelastic frictional properties", Vehicle System Dynamics, 2005, 43(sup1), pp. 199-208.

[58] Xu, N., Guo, K., Zhang, X. and Karimi, H.R., "An analytical tire model with flexible carcass for combined slips", Mathematical Problems in Engineering, Article ID 397538, 2014, 9 pp.

[59] Sarkisov, P., Prokop, G., Kubenz, J. and Popov, S., "Physical understanding of transient generation of tire lateral force and aligning torque", Tire Science and Technology, 2019,47(4), pp. 308-333.

[60] Romano, L., Sakhnevych, A., Strano, S. and Timpone, F., "A novel brush-model with flexible carcass for transient interactions", Meccanica, 2019, 54(10), pp. 1663-1679.

[61] Gipser, M., "FTire: a physically based application-oriented tyre model for use with detailed MBS and finite-element suspension models", Vehicle System Dynamics, 2005, 43(sup1), pp. 76-91.

[62] Gipser, M., "FTire–the tire simulation model for all applications related to vehicle dynamics", Vehicle System Dynamics, 2007, 45(S1), pp. 139-151.

[63] Mavros, G., Rahnejat, H. and King, P., "Investigation of steady-state tyre force and moment generation under combined longitudinal and lateral slip conditions", Proc. 18th IAVSD Symposium Kanagawa, Japan, 24 - 30 August, 2003, pp. 351 - 360.

[64] Svendenius, J., "Tire modeling and friction estimation", Doctoral Dissertation, Lund University, Sweden, 2007.

[65] Heinrich, G. and Klüppel, M., "Rubber friction, tread deformation and tire traction", Wear, 2008 265(7-8), pp. 1052-1060.

[66] Nishihara, O. and Masahiko, K., "Estimation of road friction coefficient based on the brush model", J. Dynamic Systems, Measurement, and Control, 2011, 133(4): 041006, 9 pp.

[67] Khaleghian, S., Emami, A. and Taheri, S., "A technical survey on tire-road friction estimation", Friction, 2017, 5(2), pp. 123-146.

[68] Hu, J., Rakheja, S. and Zhang, Y., "Real-time estimation of tire–road friction coefficient based on lateral vehicle dynamics", Proc. IMechE, Part D: J. Automobile Engineering, 2020, 234(10-11), pp. 2444-2457.

[69] Jaiswal, M., Mavros, G., Rahnejat, H. and King, P., "A multi-body dynamics approach for the study of critical handling manoeuvres on surfaces with uneven friction", 2nd ECCOMAS Thematic Conf. on Computational Combustion, 18-20 July 2007, Delft University of Technology, the Netherlands. Technische Universiteit Delft, TNW, 2007, 19 pp.

[70] Jansen, S. T. H., Zegelaar, P. W. A., and Pacejka, H. B., "Influence of in-plane tyre dynamics on ABS braking of a quarter vehicle model", Vehicle System Dynamics, 1999, 32, pp. 249–261.

[71] Hebden, R.G., Edwards, C. and Spurgeon, S.K., "Automotive steering control in a split-µ manoeuvre using an observer-based sliding mode controller", Vehicle System Dynamics, 2004, 41(3), pp. 181-202.

[72] Oleksowicz, S.A., Burnham, K.J., Southgate, A., McCoy, C., Waite, G., Hardwick, G., Harrington, C. and McMurran, R., "Regenerative braking strategies, vehicle safety and stability control systems: critical use-case proposals", Vehicle System Dynamics, 2013, 51(5), pp. 684-699.

[73]   Bagheri, A., Azadi, S. and Soltani, A., "A combined use of adaptive sliding mode control and unscented Kalman filter estimator to improve vehicle yaw stability", Proc. IMechE, Part K: J. Multi-body Dynamics, 2017, 231(2), pp. 388-401.

[74]   Wang, Z., Wu, J., Zhang, L. and Wang, Y., "Vehicle sideslip angle estimation for a four-wheel-independent-drive electric vehicle based on a hybrid estimator and a moving polynomial Kalman smoother", Proc. IMechE, Part K: J. Multi-body Dynamics, 2019, 233(1), pp. 125-140.

[75]   Bekker, M.G., "Mechanics of off-the-road locomotion", Proc. IMechE: Automobile Division, 1962, 16(1), pp. 25-44.

[76]   Bekker, M.G., "Introduction to terrain-vehicle systems. part i: The terrain. part ii: The vehicle", Michigan Univ. Report AD0687599, 1969, University of Michigan, Ann Arbor, USA.

[77]   Harnisch, C., "Dynamic Real-Time Simulation of the Terrain Drive of Multi-Axle Wheeled Vehicles", PhD thesis, Institute for automotive engineering, IKK, University of the federal armed forces Hamburg, Germany, 2001.

[78]   Harnisch, C., Lach, B., Jakobs, R., Troulis, M. and Nehls, O., "A New tyre–soil interaction model for vehicle simulation on deformable ground", Vehicle System Dynamics, 43 (Suppl.), 2005, pp. 384 – 394.

[79]   Sharaf, A.M., Mavros, G., Rahnejat, H. and King, P.D., "Multi-Physics Modeling Approach in All Terrain Vehicle Longitudinal Dynamics", ASME Int. Mechanical Engineering Congress and Exposition, 2006, Vol. 47675, pp. 117-126.

[80]   Sharaf, A.M., Rahnejat, H. and King, P.D., "Analysis of Handling Characteristics of All-Wheel Drive Off-Road Vehicles", Int. J. Heavy Vehicle Systems, 2008, 15(1), pp. 89-106.

[81]   Krenn, R. and Gibbesch, A., "Soft soil contact modeling technique for multi-body system simulation", Trends in Computational Contact Mechanics, 2011, pp. 135-155.

[82]   Yamashita, H., Chen, G., Ruan, Y., Jayakumar, P. and Sugiyama, H., "Hierarchical multiscale modeling of tire–soil interaction for off-road mobility simulation", J. Computational and Nonlinear Dynamics, 2019, 14(6): 061007, 11 pp.

[83]   Sullivan, C.C., Yamashita, H. and Sugiyama, H., "Reduced Order Modeling of Deformable Tire-Soil Interaction with Proper Orthogonal Decomposition", J. computational and nonlinear dynamics, 2022 17(5):051009.

[84]   Park, S., Popov, A.A. and Cole, D.J., "Influence of soil deformation on off-road heavy vehicle suspension vibration", J. Terramechanics, 2004, 41(1), pp. 41-68.

[85]   Sharaf, A.M., Mavros, G., Rahnejat, H., King, P.D. and Mohan, S.K., "Optimisation of AWD off-road vehicle performance using visco-lock devices", Int. J. Heavy Vehicle Systems, 2008, 15(2):188.

[86]   Mohan, S.K., "Tribology of differentials and traction control devices", In Tribology and Dynamics of Engine and Powertrain, Woodhead Publishing, 2010, pp. 735-772.

[87]   Senatore, C. and Sandu, C., "Torque distribution influence on tractive efficiency and mobility of off-road wheeled vehicles", J. Terramechanics, 2011, 48(5), pp. 372-383.

[88] Taheri, S., Sandu, C., Taheri, S., Pinto, E. and Gorsich, D., "A technical survey on terramechanics models for tire–terrain interaction used in modeling and simulation of wheeled vehicles", J. Terramechanics, 2015, 57, pp. 1-22.

[89] Doniselli, C., Mastinu, G. and Gobbi, M., "Aerodynamic effects on ride comfort and road holding of automobiles", Vehicle System Dynamics, 1996, 25(S1), pp. 99-125.

[90] Gillieron ,P. and Noger, C., "Contribution to the analysis of transient aerodynamic effects acting on vehicles", SAE Transactions, 2004, pp. 893-900.

[91] Hussain, K., Rahnejat, H. and Hegazy, S., "Transient vehicle handling analysis with aerodynamic interactions", Proc. IMechE, Part K: J. Multi-body Dynamics, 2007, 221(1), pp. 21-32.

[92] Winkler, N., Drugge, L., Trigell, A.S. and Efraimsson, G., "Coupling aerodynamics to vehicle dynamics in transient crosswinds including a driver model", Computers and Fluids, 2016,138, pp. 26-34.

[93] Huang, T., Gu, Z., Feng, C. and Zeng, W., "Transient aerodynamics simulations of a road vehicle in the crosswind condition coupled with the vehicle's motion", Proc. IMechE, Part D: J. Automobile Engineering, 2018, 232(5), pp. 583-598.

[94] Huang, T., Li, S., Wan, Z. and Gu, Z., "Investigation of vehicle stability under crosswind conditions based on coupling methods", Proc. IMechE, Part D: J. Automobile Engineering, 2019, 233(13), pp. 3305-3317.

[95] Brandt, A., Jacobson, B. and Sebben, S., "High speed driving stability of road vehicles under crosswinds: an aerodynamic and vehicle dynamic parametric sensitivity analysis", Vehicle System Dynamics, 2022, 60(7), pp. 2334-2357.

[96] Maltha, J., "MADYMO: A computer program for analysis and design in injury biomechanics", TNO Project. 1983, 11(3), pp. 90-92.

[97] Lupker, H.A., de Coo, P.J., Nieboer and J.J., Wismans, J.S., "Advances in MADYMO crash simulations", TNO Road-Vehicles Research Institute, Internal Report 910879, 1991, Delft, The Netherlands.

[98] Huang, Y., King, A.I. and Cavanaugh, J.M., "A MADYMO model of near-side human occupants in side impacts", J Biomechanical Engineering, 1994, 116(2), pp. 228-235.

[99] Moser, A., Steffan, H. and Kasanický, G., "The pedestrian model in PC-crash–The introduction of a multi body system and its validation", SAE transactions, 1999, pp. 794-802.

[100] Ambrósio, J. and Silva, M., "Multibody dynamics approaches for biomechanical modeling in human impact applications", In IUTAM symposium on impact biomechanics: from fundamental insights to applications, 2005, Springer, Dordrecht, The Netherlands, pp. 61-80.

[101] Ambrósio, J.A. and Kecskeméthy, A., "Multibody dynamics of biomechanical models for human motion via optimization" In Multibody Dynamics, 2007, Springer, Dordrecht, The Netherlands, pp. 245-272.

[102] Schiehlen, W., "Research trends in multibody system dynamics", Multibody System Dynamics, 2007,18(1), pp. 3-13.

[103] van Lopik, D.W. and Acar, M., "Development of a multi-body computational model of human head and neck", Proc. IMechE, Part K: J. Multi-body Dynamics, 2007, 221(2), pp. 175-197.

[104] van Lopik, D.W. and Acar M., "Dynamic verification of a multi-body computational model of human head and neck for frontal, lateral, and rear impacts", Proc. IMechE, Part K: J. Multi-body Dynamics, 2007, 221(2), pp. 199-217.

[105] Sherman, M.A., Seth, A. and Delp, S.L., "Simbody: multibody dynamics for biomedical research" Procedia IUTAM, 2011, 2, pp. 241-261.

[106] Al-Shammari, N. and Neal-Sturgess, C., "An investigation into neck injuries in simulated frontal impacts", Proc. IMechE, Part K: J. Multi-body Dynamics, 2012, 226(3), pp. 245-265.

[107] Anandan, A. and Kandavel, A., "Investigation and performance comparison of ride comfort on the created human vehicle road integrated model adopting genetic algorithm optimized proportional integral derivative control technique", Proc. IMechE, Part K: J. Multi-body Dynamics, 2020, 234(2), pp. 288-305.

[108] Pissanetzky, S., "Sparse matrix technology", Academic Press, 1984.

[109] Shabana, A. A., "Dynamics of multibody systems", Cambridge University Press, Cambridge, 2003.

[110] Orlandea, N.V., "From Newtonian dynamics to sparse Tableaux formulation and multi-body dynamics", Proc. IMechE, Part K: J. Multi-body Dynamics, 2008, 222(4), pp. 301-314.

[111] Hussein, B.A. and Shabana, A.A., "Sparse matrix implicit numerical integration of the stiff differential/algebraic equations: implementation", Nonlinear Dynamics, 2011, 65(4), pp. 369-382.

[112] Khulief, Y.A. and Shabana, A.A., "A continuous force model for the impact analysis of flexible multibody systems", Mechanism and Machine Theory, 1987, 22(3), pp. 213-224.

[113] Lankarani, H.M. and Nikravesh, P.E., "A contact force model with hysteresis damping for impact analysis of multibody systems", In ASME Int. Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Sep. 1989, Vol. 3691, pp. 45-51.

[114] Lankarani, H.M. and Nikravesh, P.E., "Continuous contact force models for impact analysis in multibody systems", Nonlinear Dynamics, 1994, 5(2), pp. 193-207.

[115] Glocker, C. and Pfeiffer, F., "Multiple impacts with friction in rigid multibody systems", Nonlinear Dynamics, 1995, 7(4), pp. 471-497.

[116] Schiehlen, W. and Seifried, R., "Three approaches for elastodynamic contact in multibody systems", Multibody System Dynamics, 2004, 12(1), pp. 1-6.

[117] Schiehlen, W., Seifried, R. and Eberhard, P., "Elastoplastic phenomena in multibody impact dynamics", Computer Methods in Applied Mechanics and Engineering, 2006, 195(50-51), pp. 6874-6890.

[118] Shabana, A.A., "Flexible multibody dynamics: review of past and recent developments", Multibody system dynamics, 1997, 1(2), pp. 189-222.

[119] Khulief, Y.A., "Modeling of impact in multibody systems: an overview", J. Computational and Nonlinear Dynamics, 2013, 8(2): 021012 (15 pp.)

[120] Corral, E., Moreno, R.G., García, M.J. and Castejón, C., "Nonlinear phenomena of contact in multibody systems dynamics: a review", Nonlinear Dynamics, 2021, 104(2), pp.1269-1295.

[121] Marques, F., Flores, P., Claro, J.C. and Lankarani, H.M., "Modeling and analysis of friction including rolling effects in multibody dynamics: a review", Multibody System Dynamics, 2019, 45(2), pp. 223-244.

[122] Pfeiffer, F., "On non-smooth multibody dynamics", Proc. IMechE, Part K: J. Multi-body Dynamics, 2012, 226(2), pp. 147-177.

[123] Rabeih, E.M. and Crolla, D.A., "Intelligent control of clutch judder and shunt phenomena in vehicle drivelines", Int. J. Vehicle Design, 1996, 17(3), pp. 318-332.

[124] Centea, D., Rahnejat, H. and Menday, M.T., "The influence of the interface coefficient of friction upon the propensity to judder in automotive clutches", Proc. IMechE, Part D: J. Automobile Engineering, 1999, 213(3), pp. 245-258.

[125] Kelly, P., Reitz, A. and Biermann, J., "The establishment of realistic multi-body clutch systems NVH targets using rig-based experimental techniques", Multi, body Dynamics: Monitoring and Simulation Techniques II, Professional Eng. Publishing, 2000:235.

[126] Centea, D., Rahnejat, H. and Menday, M.T., "Non-linear multi-body dynamic analysis for the study of clutch torsional vibrations (judder)", Applied Mathematical Modelling, 2001, 25(3), pp. 177-192.

[127] Crowther, A., Zhang, N., Liu, D.K. and Jeyakumaran, J.K., "Analysis and simulation of clutch engagement judder and stick-slip in automotive powertrain systems", Proc. IMechE, Part D: J. Automobile Engineering, 2004, 218(12), pp. 1427-1446.

[128] Flores, P., Ambrósio, J., Claro, J.P. and Lankarani, H.M., "Influence of the contact—impact force model on the dynamic response of multi-body systems", Proc. IMechE, Part K: J. Multi-body Dynamics, 2006, 220(1), pp. 21-34.

[129] Petrun, T., Flašker, J. and Kegl, M., "A friction model for dynamic analyses of multi-body systems with a fully functional friction clutch", Proc. IMechE, Part K: J. Multi-body Dynamics, 2013, 227(2), pp. 89-105.

[130] Gkinis, T., Rahmani, R. and Rahnejat, H., "Effect of clutch lining frictional characteristics on take-up judder", Proc. IMechE, Part K: J. Multi-body Dynamics, 2017, 231(3), pp. 493-503.

[131] Yang, L.K., Li, H.Y., Ahmadian, M. and Ma, B., "Analysis of the influence of engine torque excitation on clutch judder", J. Vibration and Control, 2017, 23(4), pp. 645-655.

[132] Yuan, R. and Wu, G., "Dynamic analysis of vehicle start-up judder based on elasto-plastic friction model and dry clutch maneuvering characteristic", Proc. IMechE, Part K: J. Multi-body Dynamics, 2019, 233(2), pp. 455-469.

[133] Hao H, Lu T, Zhang J, Ding W. Investigation on adaptive method of torque characteristic in dual clutch transmission during launch phase. Proc. IMechE, Part K: J. Multi-body Dynamics. 2020 Sep;234(3):568-84.

[134] Drexl, H.J., "Clutch judder—causes and counter measures", In Proc. Tech. Conf. SITEV, 1990, Apr 20, 90, pp. 7-46.

[135] Rahnejat, H., Theodossiades, S., Kelly, P. and Menday, M.T., "Drivetrain noise, vibration, and harshness", Encyclopedia of Automotive Engineering, 2014, 17:1-1.

[136] Gkinis, T., Rahmani, R. and Rahnejat, H., "Integrated thermal and dynamic analysis of dry automotive clutch linings", Applied Sciences, 2019, 9(20):4287.

[137] Dolatabadi, N., Rahmani, R., Theodossiades, S., Rahnejat, H., Blundell, G. and Bernard, G., "Tribodynamics of hydraulic actuated clutch system for engine-downsizing in heavy duty off-highway vehicles" Proc. IMechE, Part D: J. Automobile Engineering, 2019, 233(4), pp. 976-993.

[138] Hasebe, T. and Aisin Seiki, U.A., "Experimental study of reduction methods for clutch pedal vibration", SAE Tech. Pap., No. 932007, 1993.

[139] Kelly, P., Rahnejat, H. and Biermann, J.W., „C553/013/98: Multi-body dynamics investigation of clutch pedal in-cycle vibration (whoop)", In IMechE Conf. Trans., 1998, 13, pp. 167-178

[140] Fothergil, D. J. and Swierstra, N., "The Application of Non-linear Displacements Modelling Techniques to an Automotive Driveline for the Investigation of Shunt", VDI Berichte, Nr. 1007,1992, pp 163-179.

[141] Farshidianfar, A., Rahnejat, H., Ebrahimi, M. and Menday, M.T., "Low frequency torsional vibration of vehicular driveline systems in shuffle", Multi-body Dynamics: Monitoring and Simulation Techniques, 2000, pp. 269-282.

[142] Farshidianfar, A., Ebrahimi, M., Bartlett, H. and Moavenian, M., "Driveline shuffle in rear wheel vehicles", Int. J. Heavy Vehicle Systems, 2002, 9(1), pp. 76-91.

[143] Mavros, G., "Contact mechanics of tyre–road interactions and its role in vehicle shuffle", In Tribology and Dynamics of Engine and Powertrain, Woodhead Publishing, 2010, pp. 703-734

[144] Zhang, J., Chai, B. and Lu, X., "Active oscillation control of electric vehicles with two-speed transmission considering nonlinear backlash Proceedings of the Institution of Mechanical Engineers, Part K: J. Multi-body Dynamics, 2020, 234(1), pp. 116-133.

[145] Deng, C., Deng, Q., Yu, C., Zhou, L., Zhou, Y. and Hu, J., "Research on the Mechanical Mechanism of the Shuffle Problem of Electric Vehicles and the Sensitivity to Clearances", Electronics, 2022, 11(13):1935.

[146] Menday, M.T., Rahnejat and H., Ebrahimi, M., "Clonk: an onomatopoeic response in torsional impact of automotive drivelines", Proc. IMechE, Part D: J. Automobile Engineering, 1999, 213(4), pp. 349-357.

[147] Menday, M. and Ebrahimi, M., "The application of dynamic modelling using parametric design methods for an automotive driveline system", In Multi-body Dynamics Monitoring and Simulation Techniques, MEP Publication, 1997, pp. 67-79.

[148] Gnanakumarr, M., "High-energy impact-induced phenomena in driveline clonk", In Tribology and Dynamics of Engine and Powertrain, Woodhead Publishing, 2010, pp. 914-929e.

[149] Krenz, R., "Vehicle response to throttle tip in/tip out", SAE pap. No. 850967, 1985.

[150] Lee, T.H., Ahn, S.D. and Kim, Y.K., "An Investigation into Driveline Clonk Noise of Midi-Bus", In Proc. Conf. Korean Society for Noise and Vibration Engineering, The Korean Society for Noise and Vibration Engineering, 2012, pp. 658-664.

[151] Chen, H., Gao, B., Chen, H. and Gao, B., "Clutch disengagement timing control of AMT gear shift", Nonlinear Estimation and Control of Automotive Drivetrains, 2014, pp. 147-156.

[152] Turnbull, R., Miknas, O.R., Mohammadpour, M. and Rahnejat, H., "Combined experimental and flexible multi-body dynamic investigation of high-energy impact-induced driveline vibration", Proc. IMechE, Part K: J. Multi-body Dynamics, 2017, 231(1), pp. 181-193.

[153] Walker PD, Zhang N. Numerical investigations into shift transients of a dual clutch transmission equipped powertrains with multiple nonlinearities. Journal of Vibration and Control. 2015 Jun;21(8):1473-86.

[154] Lu, T., Zhou, B., Zhang, J. and Wang, X., "Clutch micro-slip control in gearshifts for dual clutch transmission", Proc. IMechE, Part K: J. Multi-body Dynamics, 2019, 233(1), pp. 43-59.

[155] Arrundale, D., Hussain, K., Rahnejat, H. and Menday, M.T., "Acoustic response of driveline pieces under impacting loads (clonk)", Proc. 31st ISATA, Dusseldorf. 1998, pp. 319-331.

[156] Theodossiades, S., Gnanakumarr, M., Rahnejat, H. and Menday, M., "Mode identification in impact-induced high-frequency vehicular driveline vibrations using an elasto-multi-body dynamics approach", Proc. IMechE, Part K: J. Multi-body Dynamics, 2004, 218(2), pp. 81-94.

[157] Craig Jr, R.R., "Structural Dynamics—An Introduction to Computer Methods", 1981, John Wiley, New York.

[158] Craig Jr, RR., "Substructure methods in vibration", *J.* Vibration and Acoustics*, 1995, 117(B), pp. 207-213.

[159] Dunn, A., Lim, T., and Houser, D., "Methods for researching gear whine in automotive transaxles", SAE Tech., pap. No. 1999-01-1768, 1999.

[160] Kim, S. J., Lee, J. Y., and Lee, S. K. Noise refinement of a vehicle by reduction of the axle gear whine noise based on structural modification using FEM and BEM. Int. J. Automotive Technol., 2007, 8(5), 605–614.

[161] Koronias, G., Theodossiades, S., Rahnejat, H. and Saunders, T., "Axle whine phenomenon in light trucks: a combined numerical and experimental investigation", Proc. IMechE, Part D: J. Automobile Engineering, 2011, 225(7), pp. 885-894.

[162] Wang, Y., Li, X., Qiao, G. and Lim, T., "Effect of component flexibility on axle system dynamics", SAE Int. J. Vehicle Dynamics, Stability, and NVH, 2017, 1(2017-01-1772), pp. 400-407.

[163] Wang, M.Y., Manoj, R. and Zhao, W., "Gear rattle modelling and analysis for automotive manual transmissions", Proc. IMechE, Part D: J. Automobile Engineering, 2001, 215(2), pp. 241-258.

[164] Brancati, R., Rocca, E. and Russo, R., "A gear rattle model accounting for oil squeeze between the meshing gear teeth", Proc. IMechE, Part D: J. Automobile Engineering, 2005, 219(9), pp. 1075-1083.

[165] Theodossiades, S., Tangasawi, O. and Rahnejat, H., "Gear teeth impacts in hydrodynamic conjunctions promoting idle gear rattle", J. Sound and Vibration, 2007, 303(3-5), pp. 632-658.

[166] De la Cruz, M., Theodossiades, S. and Rahnejat, H., "An investigation of manual transmission drive rattle", Proc. IMechE, Part K: J. Multi-body Dynamics, 2010, 224(2), pp. 167-181.

[167] Brancati, R., Rocca, E. and Russo, R., "Gear rattle reduction in an automotive driveline by the adoption of a flywheel with an innovative torsional vibration damper", Proc. IMechE, Part K: J. Multi-body Dynamics, 2019, 233(4), pp. 777-791.

[168] Padhy, S. K. and Scheldorf, G. O., "Wear and Tribodynamics of a Rolling Piston Rotary Compressor", Int. Compressor Engineering Conf. 1994, Paper 960, Purdue University, USA.

[169]- Dursunkaya, Z., Keribar, R. and Ganapathy, V., "A model of secondary motion and elastohydrodynamic skirt lubrication", J. Tribology, 1994, 116, pp. 777–785.

[170] Offner, G. and Priebsch, H.H., "Elastic body contact simulation for predicting piston slap induced noise in an IC engine", Multi-Body Dynamics: Monitoring and Simulation Techniques—II, 2000, pp. 191-206.

[171] Balakrishnan, S. and Rahnejat, H., "Isothermal transient analysis of piston skirt-to-cylinder wall contacts under combined axial, lateral and tilting motion", J. Physics, D: Applied Physics, 2005, 38(5):787.

[172] Chen, D.C., Shabana, A.A. and Rismantab-Sany, J., "Generalized and actual constraint forces in the inverse dynamic analysis of spatial flexible mechanical systems", In ASME Int. Design Eng. Tech. and Computers and Information in Engineering Conf., September 1991, 97768, pp. 469-476.

[173] Ernst, R., Schamel, A.R. and Meyer, J., "Advanced optimization techniques in valvetrain design". SAE Transactions, 1993, pp.1835-1851.

[174] Kushwaha, M., Rahnejat, H. and Johns-Rahnejat, P. M., "A tribo-multibody model for valve train kinematic analysis, quasi-statics of lubricated contact and assessment of structural integrity", ADAMS User Conf., Michigan, USA, 1997.

[175] Geradin, M., Cardona, A., Doan, D.B. and Duysens, J., "Finite element modeling concepts in multibody dynamics", Computer-aided analysis of rigid and flexible mechanical systems, 1994, pp. 233-284.

[176] Kushwaha, M., Rahnejat, H., Jin, Z.M. and Johns, P.M., "Tribo-multibody dynamic analysis of valve-train systems", In Proc. IMechE Conf. on Multi-body Dynamics: New Techniques and Applications, London, 1998, pp. 83-101.

[177] Kushwaha, M., Rahnejat, H. and Jin, Z.M., "Valve-train dynamics: a simplified tribo-elasto-multi-body analysis", Proc. IMechE, Part K: J. Multi-body Dynamics, 2000, 214(2), pp. 95-110.

[178] Vejju, K.S. and Freeman, J.S., "Multibody modeling of vehicles in a virtual prototyping environment", In ASME Int. Design Eng. Tech. Conf. and Computers and Information in Eng., 17th Sep. 1995, 17179, pp. 969-975.

[179] Boysal, A. and Rahnejat, H., "ADAMS generic engine: A tribo-elastodynamic analysis tool", In Proc. 96 ADAMS Int. Users' Conf, Mechanical Dynamics Inc., Yipsilanti, USA, May 1996, pp. 1-20.

[180] Boysal, A. and Rahnejat, H., "Torsional vibration analysis of a multi-body single cylinder internal combustion engine model", Applied Mathematical Modelling, 1997, 21(8), pp. 481-493.

[181] Zweiri, Y.H., Whidborne, J.F. and Seneviratne, L.D., "Dynamic simulation of a single-cylinder diesel engine including dynamometer modelling and friction", Proc. IMechE, Part D: J. Automobile Engineering, 1999, 213(4), pp. 391-402.

[182] Zeischka, J., Kading, D. and Crosheck, J., "Simulation of flexible engine block, crank, and valvetrain effects using DADS", Multi-body Dynamics: Monitoring and Simulation Techniques II, Bradford, 2000:169.

[183] Ma, Z.D. and Perkins, N.C., "An efficient multibody dynamics model for internal combustion engine systems", Multibody System Dynamics, 2003, 10, pp. 363-391.

[184] Okamura, H. and Morita, T., "C553/037/98: Simple modelling and analysis for crankshaft three-dimensional vibrations under firing conditions", In IMechE Conf. Trans., 1998, Vol. 13, pp. 9-26.

[185] Okamura, H. and Morita, T., "Efficient modelling and analysis for crankshaft three-dimensional vibrations under firing conditions", Proc. IMechE, Part K: J. Multi-body Dynamics, 1999, 213(1), pp. 33-44.

[186] Arrundale, D., Gupta, S. and Rahnejat, H., "Multi-body dynamics for the assessment of engine induced inertial imbalance and torsional–deflection vibration", Multi-body Dynamics: Monitoring and Simulation Techniques II, 2000, pp. 207-232.

[187] Kushwaha, M., Gupta, S., Kelly, P. and Rahnejat, H., "Elasto-multi-body dynamics of a multicylinder internal combustion engine", Proc. IMechE, Part K: J. Multi-body Dynamics, 2002, 216(4), pp. 281-293.

[188] Guzzomi, A.L., Hesterman, D.C. and Stone, B.J., "The effect of piston friction on engine block dynamics", Proc. IMechE, Part K: J. Multi-body Dynamics, 2007, 221(2), pp. 277-289.

[189] Watling, K., "Torsional vibration – analysis of crank assembly model for a reciprocating engine", Honours Thesis, The University of Western Australia, 2000.

[190] Perera, M.S., Theodossiades, S. and Rahnejat, H., "A multi-physics multi-scale approach in engine design analysis", Proc. IMechE, Part K: J. Multi-body Dynamics, 2007, 221(3), pp. 335-348.

[191] Perera, M.S., Theodossiades, S. and Rahnejat, H., "Elasto-multi-body dynamics of internal combustion engines with tribological conjunctions", Proc. IMechE, Part K: J. Multi-body Dynamics, 2010, 224(3), pp. 261-277.

[192] Craig, R. R. and Brampton, M. C. C., "Coupling of substructures for dynamics analysis", AIAA Journal, 1968, 6, pp. 1313–1319.

[193] Giakoumis, E.G., Rakopoulos, C.D. and Dimaratos, A.M., "Study of crankshaft torsional deformation under steady-state and transient operation of turbocharged diesel engines", Proc. IMechE, Part K: J. Multi-body Dynamics, 2008, 222(1), pp. 17-30.

[194] Karabulut, H., "Dynamic model of a two-cylinder four-stroke internal combustion engine and vibration treatment", Int. J. Engine Research, 2012, 13(6), pp. 616-627.

[195] Hooper, P.R., "Low noise, vibration and harshness solutions for in-line three-cylinder range extender and hybrid electric vehicles", Int. J. Engine Research, 2021, 22(2), pp. 581-591.

[196] Xu, X., Liu, Z., Wu, J., Xing, J. and Wang, X., "Misfire fault diagnosis of range extender based on harmonic analysis", Int. J. Automotive Technology, 2019, 20, pp. 99-108.

[197] Shital, P., Ghosh, C., Talwar, H., Gosain, A. and Dayal, P.S., "A study of engine mount optimisation of three-cylinder engine through multi-body dynamic simulation and its verification by vehicle measurement", SAE Tech., Pap. No. 2015-26-0126, 2015.

[198] Cavalli, M., Lavacchielli, G., Tonelli, R., Nicoletto, G. and Riva, E., "Comparison of analytical and multibody dynamic approaches in the study of a V6 engine piston", Proc. IMechE, Part K: J. Multi-body Dynamics, 2017, 231(3), pp. 420-438.

[199] Meuter, M., Offner, G. and Haase, G., "Multi-body engine simulation including elastohydrodynamic lubrication for non-conformal conjunctions", Proc. IMechE, Part K: J. Multi-body Dynamics, 2017, 231(3), pp. 457-468.

[200] Karimaei, H., Mehrgou, M. and Chamani, H.R., "Optimisation of torsional vibration system for a heavy-duty inline six-cylinder diesel engine", Proc. IMechE, Part K: J. Multi-body Dynamics, 2019, 233(3), pp. 642-656.